Mean electromotive force due to turbulence of a conducting fluid in the presence of mean flow

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(Received 14 December 2005; revised manuscript received 3 March 2006; published 24 May 2006)

The mean electromotive force caused by turbulence of an electrically conducting fluid, which plays a central part in mean-field electrodynamics, is calculated for a rotating fluid. Going beyond most of the investigations on this topic, an additional mean motion in the rotating frame is taken into account. One motivation for our investigation originates from a planned laboratory experiment with a Ponomarenko-type dynamo. In view of this application the second-order correlation approximation is used. The investigation is of high interest in astrophysical context, too. Some contributions to the mean electromotive are revealed which have not been considered so far, in particular contributions to the α effect and related effects due to the gradient of the mean velocity. Their relevance for dynamo processes is discussed. In a forthcoming paper the results reported here will be specified to the situation in the laboratory and partially compared with experimental findings.

DOI: 10.1103/PhysRevE.73.056311 PACS number(s): 47.65.−d, 47.27.−i

I. INTRODUCTION

In mean-field electrodynamics of turbulent fluids the mean electromagnetic fields follow Maxwell's equations. The turbulence, however, gives rise to a mean electromotive force, which occurs in Ohm's law and, consequently, in the induction equation. This mean electromotive force, which is crucial in the theory of cosmic magnetic fields and dynamos as well as in other fields, has been an objective of many investigations. It has been calculated in specific approximations for different forms of turbulence on a rotating body under the assumption of zero mean motion of the fluid in the rotating frame, see, e.g., Refs. [1–10]. In a few cases also the effect of a mean motion has been studied. There are some rather general results of that kind, e.g., Refs. [1,2], the application of which requires however further elaboration. The more detailed results derived recently, Refs. [11–14], are not in convincing agreement with each other.

By this reason we have again dealt with the mean electromotive force in a rotating turbulent fluid in the presence of a mean motion. The primary motivation for dealing with this topic was to find estimates of the effects of turbulence in a laboratory experiment with a screw dynamo as proposed by Ponomarenko [15], which is under preparation in the Institute for Continuous Media Mechanics in Perm; see Refs. [16–19]. Moreover the results are of high interest for astrophysical applications, for instance in view of the possibility of the "$\mathbf{W}\times\mathbf{J}$ dynamo," which has been proposed recently [14,20].

In this paper the mean electromotive force is considered in the presence of a more or less arbitrary mean flow, and in a forthcoming paper [21] we will specify the results and discuss them in view of the situation in the experimental device. (For a first, very short report on this topic see Ref. [22].) In Sec. II of this paper we describe the general framework of our investigation. In Sec. III we explain some general aspects of our view on the problem and use basic symmetry laws to draw conclusions concerning the structure of the mean electromotive force, that is, concerning its dependence on vectorial and tensorial quantities that characterize the turbulence and the mean motion. In order to determine the mean electromotive force completely, we introduce in Sec. IV specific approximations, in particular some kind of second-order approximation, and calculate all of its coefficients in their dependence on the intensity of the turbulence and related parameters. Finally in Sec. V we discuss our results in general terms, compare them with those of other investigations and point out their consequences for dynamo processes.

II. MEAN-FIELD MAGNETOHYDRODYNAMICS

A. Electromagnetic fundamentals

Let us assume that the behavior of the magnetic field $\mathbf{B}$ in an electrically conducting fluid is governed by the induction equation

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{U} \times \mathbf{B}) - \eta \nabla^2 \mathbf{B} = \mathbf{0}, \quad \nabla \cdot \mathbf{B} = 0. \quad (1)$$

$\mathbf{U}$ is the velocity and η the magnetic diffusivity of the fluid, the latter being considered as constant.

Following the lines of mean-field electrodynamics (see, e.g., Refs. [2,5]) we define mean magnetic and velocity fields, $\overline{\mathbf{B}}$ and $\overline{\mathbf{U}}$, as averages over space or time scales larger than those of the turbulence. We call $\mathbf{B}-\overline{\mathbf{B}}$ and $\mathbf{U}-\overline{\mathbf{U}}$ simply "fluctuations" and denote them by $\mathbf{b}$ and $\mathbf{u}$, respectively. We further assume that the Reynolds averaging rules apply. Taking the average of Eq. (1) we obtain the mean-field induction equation

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$$\partial_t \bar{\mathbf{B}} - \nabla \times (\bar{\mathbf{U}} \times \bar{\mathbf{B}} + \boldsymbol{\mathcal{E}}) - \eta \nabla^2 \bar{\mathbf{B}} = \mathbf{0}, \quad \nabla \cdot \bar{\mathbf{B}} = 0, \quad (2)$$

where $\boldsymbol{\mathcal{E}}$ is the mean electromotive force due to the fluctuations of magnetic field and velocity,

$$\boldsymbol{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}. \quad (3)$$

The equation for $\mathbf{b}$ resulting from (1) and (2) allows us to conclude that $\boldsymbol{\mathcal{E}}$ can be considered as a functional of $\bar{\mathbf{U}}$, $\mathbf{u}$, and $\bar{\mathbf{B}}$, which is linear in $\bar{\mathbf{B}}$. Furthermore $\boldsymbol{\mathcal{E}}$ in a given point in space and time depends on $\bar{\mathbf{U}}$, $\mathbf{u}$, and $\bar{\mathbf{B}}$ not only in this point but also on their behaviors in a certain neighborhood of this point. We assume that $\boldsymbol{\mathcal{E}}$ has no part independent of $\bar{\mathbf{B}}$, that is, it is not only linear but also homogeneous in $\bar{\mathbf{B}}$. We further accept the frequently used assumption that $\bar{\mathbf{B}}$ varies only weakly in space and time so that $\boldsymbol{\mathcal{E}}$ in a given point depends on $\bar{\mathbf{B}}$ only via its components and their first spatial derivatives in this point. Hence, $\boldsymbol{\mathcal{E}}$ can be represented in the form

$$\mathcal{E}_i = a_{ij}\bar{B}_j + b_{ijk}\partial \bar{B}_j/\partial x_k, \quad (4)$$

where the tensors a_{ij} and b_{ijk} are averaged quantities determined by $\bar{\mathbf{U}}$ and $\mathbf{u}$. Here and in the following a Cartesian coordinate system (x_1, x_2, x_3) is used and the summation convention is adopted. Relation (4) is equivalent to

$$\boldsymbol{\mathcal{E}} = -\boldsymbol{\alpha} \circ \bar{\mathbf{B}} - \boldsymbol{\gamma} \times \bar{\mathbf{B}} - \beta \circ (\nabla \times \bar{\mathbf{B}}) - \boldsymbol{\delta} \times (\nabla \times \bar{\mathbf{B}}) - \boldsymbol{\kappa} \circ (\nabla \bar{\mathbf{B}})^{(s)}; \quad (5)$$

see, e.g., Ref. [4] or Ref. [10]. Here $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are symmetric tensors of the second rank, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are vectors, and $\boldsymbol{\kappa}$ is a tensor of the third rank, all depending on $\bar{\mathbf{U}}$ and $\mathbf{u}$ only. Further $(\nabla \bar{\mathbf{B}})^{(s)}$ is the symmetric part of the gradient tensor of $\bar{\mathbf{B}}$, i.e., $(\nabla \bar{\mathbf{B}})^{(s)}_{ij} = \frac{1}{2}(\partial \bar{B}_i/\partial x_j + \partial \bar{B}_j/\partial x_i)$. Notations like $\alpha \circ \bar{\mathbf{B}}$ are used in the sense of $(\boldsymbol{\alpha} \circ \bar{\mathbf{B}})_i = \alpha_{ij}\bar{B}_j$, and $\boldsymbol{\kappa} \circ (\nabla \bar{\mathbf{B}})^{(s)}$ is defined by $(\boldsymbol{\kappa} \circ (\nabla \bar{\mathbf{B}})^{(s)})_i = \kappa_{ijk}(\nabla \bar{\mathbf{B}})^{(s)}_{jk}$.

The term with $\boldsymbol{\alpha}$ in (5) describes the α effect, which is in general anisotropic, that with $\boldsymbol{\gamma}$ a transport of mean magnetic flux by the turbulence. The terms with $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ can be interpreted by introducing a modified magnetic diffusivity, again in general anisotropic. The induction effects which correspond to these terms are usually accompanied by such described by the term $\boldsymbol{\kappa}$, which allows no simple independent interpretation. More details will be explained in Secs. V B–V D.

The quantities $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ are connected with a_{ij} and b_{ijk} by

$$\alpha_{ij} = -\tfrac{1}{2}(a_{ij} + a_{ji}), \quad \gamma_i = \tfrac{1}{2}\epsilon_{ijk}a_{jk},$$

$$\beta_{ij} = \tfrac{1}{4}(\epsilon_{ikl}b_{jkl} + \epsilon_{jkl}b_{ikl}), \quad \delta_i = \tfrac{1}{4}(b_{jji} - b_{jij}),$$

$$\kappa_{ijk} = -\tfrac{1}{2}(b_{ijk} + b_{ikj}). \quad (6)$$

B. Momentum balance

We will consider the situation as described so far in a rotating frame of reference and restrict our attention to an incompressible fluid. The fluid velocity $\mathbf{U}$ is assumed to satisfy the momentum balance and the incompressibility condition in the form

$$\partial_t \mathbf{U} + (\mathbf{U} \cdot \nabla)\mathbf{U} = \varrho^{-1} \nabla p + \nu \nabla^2 \mathbf{U} - 2\boldsymbol{\Omega} \times \mathbf{U} + \mathbf{f},$$

$$\nabla \cdot \mathbf{U} = 0. \quad (7)$$

Here ϱ is the mass density and ν is the kinematic viscosity of the fluid, p is the hydrodynamic pressure including the centrifugal pressure, $\boldsymbol{\Omega}$ is the angular velocity responsible for the Coriolis force, and $\mathbf{f}$ is an artificial external force, which should mimic the cause of the turbulence. Any influence of the magnetic field on the fluid motion is ignored.

III. THE STRUCTURE OF THE MEAN ELECTROMOTIVE FORCE $\boldsymbol{\mathcal{E}}$

A. Change to a proper frame of reference

Let us now focus our attention on the electromotive force $\boldsymbol{\mathcal{E}}$ in a given point, consider the mean motion as independent of time and specify the frame of reference in which (7) applies such that $\bar{\mathbf{U}} = \mathbf{0}$ in this point. $\boldsymbol{\mathcal{E}}$ must be interpreted as a force on charged particles rather than a part of the electric field. Therefore the result for $\boldsymbol{\mathcal{E}}$ obtained in a given frame, understood as a vector with the usual transformation properties, applies then also in any other frame; see also Ref. [21]. Remaining in the frame specified such that $\mathbf{U} = \mathbf{0}$ in the considered point we introduce the simplifying assumption that in the neighborhood of this point relevant for the determination of $\boldsymbol{\mathcal{E}}$ the mean velocity $\bar{\mathbf{U}}$ varies only weakly. More precisely, we assume that it can be represented there with respect to the frame specified above in the form $\bar{U}_i = U_{ij}x_j$ with U_{ij} being constant, where (x_1, x_2, x_3) means a new Cartesian coordinate system defined in the rotating frame such that $x_1 = x_2 = x_3 = 0$ in the point considered.

B. Homogeneous background turbulence

We further assume until further notice that the turbulent fluctuations $\mathbf{u}$ deviate from a homogeneous isotropic mirror-symmetric turbulence only as a consequence of the Coriolis force defined by $\boldsymbol{\Omega}$ and of the gradient of the mean fluid velocity, that is, the gradient tensor $\nabla \bar{\mathbf{U}}$ given by $(\nabla \bar{\mathbf{U}})_{ij} = \partial \bar{U}_i/\partial x_j$, or $(\nabla \bar{\mathbf{U}})_{ij} = U_{ij}$. For particular purposes we split $\nabla \bar{\mathbf{U}}$ into its symmetric and antisymmetric parts. The symmetric one is the rate of strain tensor, D, given by $D_{ij} = \frac{1}{2}(\partial \bar{U}_i/\partial x_j + \partial \bar{U}_j/\partial x_i)$. It describes the deforming motion close to the point considered. Due to the incompressibility of the fluid we have $\nabla \cdot \bar{\mathbf{U}} = 0$ and therefore $D_{ii} = 0$. The antisymmetric part, A, given by $A_{ij} = \frac{1}{2}(\partial \bar{U}_i/\partial x_j - \partial \bar{U}_j/\partial x_i)$, corresponds to a rigid body rotation of the fluid close to this point. We may represent it according to $A_{ij} = -\frac{1}{2}\epsilon_{ijl}W_l$ by the vector $\mathbf{W} = \nabla \times \bar{\mathbf{U}}$.

In order to prepare conclusions concerning the structure of $\boldsymbol{\mathcal{E}}=\overline{\mathbf{u}\times\mathbf{b}}$ we first recall symmetry properties of the equations (1) and (7) governing $\mathbf{B}$ and $\mathbf{U}$. If these equations are satisfied with given $\mathbf{B}$, $\mathbf{U}$, ∇p, $\boldsymbol{\Omega}$, and $\mathbf{f}$, they are, too, with other $\mathbf{B}$, $\mathbf{U}$, ∇p, $\boldsymbol{\Omega}$, and $\mathbf{f}$ derived from the given ones by a rotation about any axis running, e.g., through $\mathbf{x}=\mathbf{0}$. Likewise they are satisfied with $\mathbf{B}$, $\mathbf{U}$, ∇p, $\boldsymbol{\Omega}$, and $\mathbf{f}$ derived from the given ones by reflecting them at a plane, e.g., one containing $\mathbf{x}=\mathbf{0}$ and, in addition, changing the signs of $\mathbf{B}$ and $\boldsymbol{\Omega}$. These two properties apply analogously to consequences drawn from these equations, in particular to the equations governing $\mathbf{u}$ and $\mathbf{b}$. The first property, connected with the rotation of fields, leads to the conclusion that the tensors a_{ij} and b_{ijk}, which occur in (4), and therefore also $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ cannot contain any construction elements other than the isotropic tensors δ_{lm} and ϵ_{lmn}, the vectors $\boldsymbol{\Omega}$ and $\mathbf{W}$ and the tensor $\mathbf{D}$. Note that the force $\mathbf{f}$, which is assumed to cause a homogeneous isotropic turbulence, cannot introduce other than isotropic quantities. The second property, connected with reflection, is often described by considering $\mathbf{U}$, ∇p, and $\mathbf{f}$ as polar and $\mathbf{B}$ and $\boldsymbol{\Omega}$ as axial vector fields. By contrast to polar vectors the axial ones show specific sign changes of their components under reflection of the coordinate system. We adopt this concept and distinguish between "true" and "pseudo" scalars, vectors and tensors, where we call polar and axial vectors simply "true" and "pseudo" vectors, respectively. Then a_{ij} and b_{ijk} must be pseudotensors. This implies that $\boldsymbol{\alpha}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$ are also pseudoquantities but $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ true quantities.

Let us consider first $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$. The mentioned construction elements δ_{lm}, ϵ_{lmn}, $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$ allow us neither to build a pseudotensor of the second rank nor a true vector. That is, we have

$$\alpha_{ij}=0, \quad \gamma_i=0. \tag{8}$$

In contrast to this there are several nonzero contributions to β_{ij}, δ_i, and κ_{ijk}. For the sake of simplicity we give only those of them which are linear in $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$, that is,

$$\beta_{ij}=\beta^{(0)}\delta_{ij}+\beta^{(D)}D_{ij}, \quad \delta_i=\delta^{(\Omega)}\Omega_i+\delta^{(W)}W_i,$$

$$\kappa_{ijk}=\tfrac{1}{2}\kappa^{(\Omega)}(\Omega_j\delta_{ik}+\Omega_k\delta_{ij})+\tfrac{1}{2}\kappa^{(W)}(W_j\delta_{ik}+W_k\delta_{ij}) + \kappa^{(D)}(\epsilon_{ijl}D_{kl}+\epsilon_{ikl}D_{jl}). \tag{9}$$

Here $\beta^{(0)}$, $\beta^{(D)}$, $\delta^{(\Omega)},\ldots$ are coefficients determined by $\mathbf{u}$ but independent of $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$. Because of $\nabla\cdot\overline{\mathbf{B}}=0$ terms of κ_{ijk} containing δ_{jk} would not contribute to $\boldsymbol{\mathcal{E}}$ and may therefore be dropped.

As a consequence of (8) and (9) we have

$$\begin{aligned}\boldsymbol{\mathcal{E}}=&-\beta^{(0)}\nabla\times\overline{\mathbf{B}}-\beta^{(D)}\mathbf{D}\circ(\nabla\times\overline{\mathbf{B}})\\&-(\delta^{(\Omega)}\boldsymbol{\Omega}+\delta^{(W)}\mathbf{W})\times(\nabla\times\overline{\mathbf{B}})\\&-(\kappa^{(\Omega)}\boldsymbol{\Omega}+\kappa^{(W)}\mathbf{W})\circ(\nabla\overline{\mathbf{B}})^{(s)}-\kappa^{(D)}\hat{\boldsymbol{\kappa}}(\mathbf{D})\circ(\nabla\overline{\mathbf{B}})^{(s)},\end{aligned} \tag{10}$$

where $\hat{\boldsymbol{\kappa}}(D)$ is a tensor of the third rank defined by $\hat{\kappa}_{ijk}=\epsilon_{ijl}D_{lk}+\epsilon_{ikl}D_{lj}$. Quantities like $\beta^{(0)},\beta^{(D)},\ldots,\kappa^{(D)}$ are called "mean-field coefficients" in the following.

The $\beta^{(0)}$ and $\beta^{(D)}$ terms in (10) make that the mean-field diffusivity deviates from the original magnetic diffusivity η of the fluid. Due to the $\beta^{(0)}$ term the mean-field diffusivity turns into $\eta+\beta^{(0)}$, due to the $\beta^{(D)}$ term it becomes anisotropic. The $\delta^{(\Omega)}$ and $\delta^{(W)}$ terms, too, can be discussed as contributions to the mean-field diffusivity. They lead to skew-symmetric contributions to the diffusivity tensor. In another context the effect described by the $\delta^{(\Omega)}$ term has been called "$\boldsymbol{\Omega}\times\mathbf{J}$ effect." It has been shown that this effect in combination with a differential rotation, here a dependence of $\boldsymbol{\Omega}$ on r, is able to establish a dynamo; see Refs. [23–26]. The $\delta^{(W)}$ term describes an effect analogous to the $\boldsymbol{\Omega}\times\mathbf{J}$ effect, which has been revealed only recently [14]. We call it "$\mathbf{W}\times\mathbf{J}$ effect." It occurs however even in the absence of the Coriolis force, only as consequence of a shear in the mean motion. We will discuss the $\delta^{(\Omega)}$ and $\delta^{(W)}$ effects as well as the $\kappa^{(\Omega)}$, $\kappa^{(W)}$, and $\kappa^{(D)}$ effects in more detail in Sec. V D.

C. Inhomogeneous background turbulence

Let us now relax the assumption that the original turbulence is homogeneous and isotropic. We admit an inhomogeneity and an anisotropy due to a gradient of a quantity like the turbulence intensity and introduce a vector $\mathbf{g}$ in the direction of this gradient, e.g., by setting $\nabla\overline{u^2}=\mathbf{g}\overline{u^2}$, with $\overline{u^2}$ derived from the turbulent velocity $\mathbf{u}$. Then we have to add $\mathbf{g}$ to the above-mentioned construction elements of $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\kappa}$. As a consequence $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ can well be nonzero. For the sake of simplicity we assume that the influence of $\mathbf{g}$ on these quantities is so weak that they are at most of first order in $\mathbf{g}$. We have then

$$\begin{aligned}\alpha_{ij}=&\,\alpha_1^{(\Omega)}(\mathbf{g}\cdot\Omega)\delta_{ij}+\alpha_2^{(\Omega)}(g_i\Omega_j+g_j\Omega_i)+\alpha_1^{(W)}(\mathbf{g}\cdot\mathbf{W})\delta_{ij}\\&+\alpha_2^{(W)}(g_iW_j+g_jW_i)+\alpha^{(D)}(\epsilon_{ilm}D_{jl}+\epsilon_{jlm}D_{il})g_m,\\ \gamma_i=&\,\gamma^{(0)}g_i+\gamma^{(\Omega)}\epsilon_{ilm}g_l\Omega_m+\gamma^{(W)}\epsilon_{ilm}g_lW_m+\gamma^{(D)}g_jD_{ij},\end{aligned} \tag{11}$$

whereas (9) remains unchanged.

Consequently $\boldsymbol{\mathcal{E}}$ takes the form

$$\begin{aligned}\boldsymbol{\mathcal{E}}=&-\alpha_1^{(\Omega)}(\mathbf{g}\cdot\boldsymbol{\Omega})\overline{\mathbf{B}}-\alpha_2^{(\Omega)}((\boldsymbol{\Omega}\cdot\overline{\mathbf{B}})\mathbf{g}+(\mathbf{g}\cdot\overline{\mathbf{B}})\boldsymbol{\Omega})\\&-\alpha_1^{(W)}(\mathbf{g}\cdot\mathbf{W})\overline{\mathbf{B}}-\alpha_2^{(W)}((\mathbf{W}\cdot\overline{\mathbf{B}})\mathbf{g}+(\mathbf{g}\cdot\overline{\mathbf{B}})\mathbf{W})\\&-\alpha^{(D)}\hat{\alpha}(\mathbf{g},\mathbf{D})\circ\overline{\mathbf{B}}\\&-(\gamma^{(0)}\mathbf{g}+\gamma^{(\Omega)}\mathbf{g}\times\boldsymbol{\Omega}+\gamma^{(W)}\mathbf{g}\times\mathbf{W}+\gamma^{(D)}\mathbf{g}\circ\mathbf{D})\\&\times\overline{\mathbf{B}}-\beta^{(0)}\nabla\times\overline{\mathbf{B}}-\beta^{(D)}\mathbf{D}\circ(\nabla\times\overline{\mathbf{B}})\\&-(\delta^{(\Omega)}\boldsymbol{\Omega}+\delta^{(W)}\mathbf{W})\times(\nabla\times\overline{\mathbf{B}})\\&-(\kappa^{(\Omega)}\boldsymbol{\Omega}+\kappa^{(W)}\mathbf{W})\circ(\nabla\overline{\mathbf{B}})^{(s)}-\kappa^{(D)}\hat{\boldsymbol{\kappa}}(\mathbf{D})\circ(\nabla\overline{\mathbf{B}})^{(s)},\end{aligned} \tag{12}$$

where $\hat{\boldsymbol{\alpha}}(\mathbf{g},\mathbf{D})$ is a symmetric tensor defined by $\hat{\alpha}_{ij}=(\epsilon_{ilm}D_{lj}+\epsilon_{jlm}D_{li})g_m$.

IV. CALCULATION OF THE MEAN ELECTROMOTIVE FORCE $\boldsymbol{\mathcal{E}}$

A. Basic equations and approximations

Our considerations on the structure of the electromotive force $\boldsymbol{\mathcal{E}}$ did not provide us with results for the coefficients $\hat{\alpha}_1^{(\Omega)}, \hat{\alpha}_2^{(\Omega)}, \ldots, \kappa^{(D)}$ showing their dependence on the intensity or other parameters of the turbulent flow. To calculate these coefficients we start again from the induction equation (1) and the momentum balance in the form (7), both related to the moving frame of reference defined above. From Eq. (1) and its mean-field version (2) and from Eq. (7) and the corresponding mean-field version we derive the equations governing the magnetic and velocity fluctuations $\mathbf{b}$ and $\mathbf{u}$,

$$\partial_t \mathbf{b} - \eta \nabla^2 \mathbf{b} = \nabla \times (\bar{\mathbf{U}} \times \mathbf{b} + \mathbf{u} \times \bar{\mathbf{B}} + (\mathbf{u} \times \mathbf{b})'),$$

$$\nabla \cdot \mathbf{b} = 0,$$

$$\partial_t \mathbf{u} - \nu \nabla^2 \mathbf{u} = -\varrho^{-1} \nabla p' - 2\boldsymbol{\Omega} \times \mathbf{u} - (\bar{\mathbf{U}} \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\bar{\mathbf{U}} - ((\mathbf{u} \cdot \nabla)\mathbf{u})' + \mathbf{f}',$$

$$\nabla \cdot \mathbf{u} = 0, \tag{13}$$

where $(\mathbf{u}\times\mathbf{b})' = \mathbf{u}\times\mathbf{b} - \overline{\mathbf{u}\times\mathbf{b}}$ and $((\mathbf{u}\cdot\nabla)\mathbf{u})' = (\mathbf{u}\cdot\nabla)\mathbf{u} - \overline{(\mathbf{u}\cdot\nabla)\mathbf{u}}$. In view of the calculation of the electromotive force $\boldsymbol{\mathcal{E}}$ in the point $\mathbf{x}=\mathbf{0}$ of the comoving frame of reference we consider these equations in some surroundings of this point. Adopting the assumptions introduced above on sufficiently weak variations of $\bar{\mathbf{B}}$ and $\bar{\mathbf{U}}$ in space and time we set

$$\bar{B}_i = B_i + B_{ij} x_j, \quad \bar{U}_i = U_{ij} x_j, \tag{14}$$

with B_i, B_{ij}, and U_{ij} being constants and satisfying $U_{ii} = B_{ii} = 0$.

We restrict our calculation on the case in which the influences of both the Coriolis force and the shear of the mean motion on $\mathbf{u}$ and $\mathbf{b}$ are only weak. We introduce the expansions

$$\mathbf{u} = \mathbf{u}^{(0)} + \mathbf{u}^{(1)} + \cdots, \quad \mathbf{b} = \mathbf{b}^{(0)} + \mathbf{b}^{(1)} + \cdots, \tag{15}$$

where $\mathbf{u}^{(0)}$ and $\mathbf{b}^{(0)}$ are independent of $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$, further $\mathbf{u}^{(1)}$ and $\mathbf{b}^{(1)}$ are of first order and all contributions indicated by $\cdots$ are of higher order in these quantities. In that sense we have

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}^{(0)} + \boldsymbol{\mathcal{E}}^{(1)} + \cdots,$$

$$\boldsymbol{\mathcal{E}}^{(0)} = \boldsymbol{\mathcal{E}}^{(00)}, \quad \boldsymbol{\mathcal{E}}^{(1)} = \boldsymbol{\mathcal{E}}^{(10)} + \boldsymbol{\mathcal{E}}^{(01)}, \ldots,$$

$$\boldsymbol{\mathcal{E}}^{(\alpha\beta)} = \overline{\mathbf{u}^{(\alpha)} \times \mathbf{b}^{(\beta)}}. \tag{16}$$

In the following we restrict our attention on the case in which $\boldsymbol{\mathcal{E}}$ is linear in $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$, that is, on the terms $\boldsymbol{\mathcal{E}}^{(0)}$ and $\boldsymbol{\mathcal{E}}^{(1)}$ in this expansion of $\boldsymbol{\mathcal{E}}$.

We assume that both $\mathbf{u}$ and $\mathbf{b}$ are small enough so that the second-order correlation approximation (SOCA) applies, sometimes also labeled as first-order smoothing approximation (FOSA), which is often used in the astrophysical context. So we conclude from (13) that

$$\partial_t \mathbf{u}^{(0)} - \nu \nabla^2 \mathbf{u}^{(0)} = -\varrho^{-1} \nabla p^{(0)} + ((\mathbf{u}^{(0)} \cdot \nabla)\mathbf{u}^{(0)})' + \mathbf{f}^{(0)},$$

$$\nabla \cdot \mathbf{u}^{(0)} = 0,$$

$$\partial_t \mathbf{u}^{(1)} - \nu \nabla^2 \mathbf{u}^{(1)} = -\varrho^{-1} \nabla p^{(1)} - (\bar{\mathbf{U}} \cdot \nabla)\mathbf{u}^{(0)} - (\mathbf{u}^{(0)} \cdot \nabla)\bar{\mathbf{U}} - 2\boldsymbol{\Omega} \times \mathbf{u}^{(0)},$$

$$\nabla \cdot \mathbf{u}^{(1)} = 0,$$

$$\partial_t \mathbf{b}^{(0)} - \eta \nabla^2 \mathbf{b}^{(0)} = \nabla \times (\mathbf{u}^{(0)} \times \bar{\mathbf{B}}),$$

$$\nabla \cdot \mathbf{b}^{(0)} = 0,$$

$$\partial_t \mathbf{b}^{(1)} - \eta \nabla^2 \mathbf{b}^{(1)} = \nabla \times (\mathbf{u}^{(1)} \times \bar{\mathbf{B}} + \bar{\mathbf{U}} \times \mathbf{b}^{(0)}),$$

$$\nabla \cdot \mathbf{b}^{(1)} = 0. \tag{17}$$

We consider the turbulent fluid motion in the limit of zero Coriolis force and zero shear, that is $\mathbf{u}^{(0)}$, as given. In deriving (17) we have assumed that the force $\mathbf{f}'$ does not depend on $\boldsymbol{\Omega}$, $\mathbf{W}$ or $\mathbf{D}$ and therefore possesses no other contributions than $\mathbf{f}^{(0)}$. Following the traditional second-order approximation as used in situations in which $\mathbf{u}$ is given we have neglected $(\mathbf{u}^{(0)}\times\mathbf{b}^{(0)})'$ in comparison with $\mathbf{u}^{(0)}\times\bar{\mathbf{B}}$. In the same spirit we have further neglected $((\mathbf{u}^{(0)}\cdot\nabla)\mathbf{u}^{(1)})' + ((\mathbf{u}^{(1)}\cdot\nabla)\mathbf{u}^{(0)})'$ in comparison with $(\bar{\mathbf{U}}\cdot\nabla)\mathbf{u}^{(0)} + (\mathbf{u}^{(0)}\cdot\nabla)\bar{\mathbf{U}} - 2\boldsymbol{\Omega}\times\mathbf{u}^{(0)}$ and $(\mathbf{u}^{(0)}\times\mathbf{b}^{(1)})' + (\mathbf{u}^{(1)}\times\mathbf{b}^{(0)})'$ in comparison with $\mathbf{u}^{(1)}\times\bar{\mathbf{B}} + \mathbf{u}^{(1)}\times\mathbf{b}^{(0)}$. The justification for these omissions must be checked in all applications.

In Sec. II A we have introduced the assumption that $\boldsymbol{\mathcal{E}}$ has no contribution independent of $\bar{\mathbf{B}}$. In the second-order correlation approximation this assumption is automatically satisfied, a contribution of this kind cannot occur. The second-order correlation approximation in the above sense also excludes any kind of magnetohydrodynamic turbulence. In the limit of small $\bar{\mathbf{B}}$ the turbulence is purely hydrodynamic.

B. Fourier representation

We will carry out some of our calculations in the Fourier space. The Fourier transformation is defined in the form

$$Q(\mathbf{x},t) = \int\int \hat{Q}(\mathbf{k},\omega) \exp(i(\mathbf{k}\cdot\mathbf{x} - \omega t)) d^3k\, d\omega, \tag{18}$$

where the integrations are over all $\mathbf{k}$ and ω.

Let us consider the two-point correlation tensor ϕ_{ij} for two vector fields $\mathbf{v}$ and $\mathbf{w}$ defined by

$$\phi_{ij}(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) = \langle v_i(\mathbf{x}_1, t_1) w_j(\mathbf{x}_2, t_2) \rangle. \tag{19}$$

Here and in what follows the notation $\langle X \rangle$ is used in the same sense as $\bar{X}$. Following Ref. [27] we consider ϕ_{ij} also as a function of the variables

$$\mathbf{R} = (\mathbf{x}_1 + \mathbf{x}_2)/2, \quad \mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2,$$

$$T = (t_1 + t_2)/2, \quad t = t_1 - t_2 \tag{20}$$

and write then

$$\phi_{ij}(\mathbf{R},T;\mathbf{r},t) = \langle v_i(\mathbf{R}+\mathbf{r}/2,T+t/2)w_i(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle. \tag{21}$$

Clearly we have

$$\phi_{ij}(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2) = \int\int\int\int \langle \hat{v}_i(\mathbf{k}_1,\omega_1)\hat{w}_j(\mathbf{k}_1,\omega_1)\rangle \times \exp[i(\mathbf{k}_1\mathbf{x}_1 + \mathbf{k}_2\mathbf{x}_2 - \omega_1 t_1 - \omega_2 t_2)] \times d^3k_1 d\omega_1 d^3k_2 d\omega_2. \tag{22}$$

In addition to (20) we introduce

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2, \quad \mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2,$$

$$\Omega = \omega_1 + \omega_2, \quad \omega = (\omega_1 - \omega_2)/2 \tag{23}$$

and arrive so at

$$\phi_{ij}(\mathbf{R},T;\mathbf{r},t) = \int\int \tilde{\phi}_{ij}(\mathbf{R},T;\mathbf{k},\omega)\exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)d^3k d\omega \tag{24}$$

with

$$\tilde{\phi}_{ij}(\mathbf{R},T;\mathbf{k},\omega) = \int\int \langle \hat{v}_i(\mathbf{k}+\mathbf{K}/2,\omega+\Omega/2) \times \hat{w}_j(-\mathbf{k}+\mathbf{K}/2,-\omega+\Omega/2)\rangle \times \exp[i(\mathbf{K}\mathbf{R} - \Omega T)]d^3K d\Omega. \tag{25}$$

In the sense of (21) we introduce in view of the following calculations:

$$\chi_{ij}(\mathbf{R},T;\mathbf{r},t) = \langle u_i(\mathbf{R}+\mathbf{r}/2,T+t/2)b_j(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle,$$

$$v_{ij}(\mathbf{R},T;\mathbf{r},t) = \langle u_i(\mathbf{R}+\mathbf{r}/2,T+t/2)u_j(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle, \tag{26}$$

and denote the quantities that correspond to $\tilde{\phi}_{ij}$ by $\tilde{\chi}_{ij}$ and $\tilde{v}_{ij}$, respectively. We extend these definitions to cases where u_i is replaced by $u_i^{(\alpha)}$, and b_j or u_j by $b_j^{(\beta)}$ or $u_j^{(\beta)}$, and use then the notations $\chi_{ij}^{(\alpha\beta)}$, $v_{ij}^{(\alpha\beta)}$, $\tilde{\chi}_{ij}^{(\alpha\beta)}$, and $\tilde{v}_{ij}^{(\alpha\beta)}$. For the correlation tensors $v_{ij}^{(00)}$ and $\tilde{v}_{ij}^{(00)}$ of the background turbulence we write simply $v_{ij}^{(0)}$ and $\tilde{v}_{ij}^{(0)}$. Since $\nabla\cdot\mathbf{u}^{(0)}=0$ we have

$$k_j\tilde{v}_{ji}^{(0)} = \frac{i}{2}\nabla_j\tilde{v}_{ji}^{(0)}, \quad k_j\tilde{v}_{ij}^{(0)} = -\frac{i}{2}\nabla_j\tilde{v}_{ij}^{(0)}. \tag{27}$$

If, as here, both $\mathbf{R}$ and $\mathbf{r}$ occur in arguments, ∇_i must be understood as $\partial/\partial R_i$.

C. Preparations for the calculation of $\boldsymbol{\mathcal{E}}$

Returning now to the electromotive force $\boldsymbol{\mathcal{E}}$ we note first that

$$\mathcal{E}_i(\mathbf{R},T) = \epsilon_{ilm}\chi_{lm}(\mathbf{R},T;0,0) = \epsilon_{ilm}\int\int \tilde{\chi}_{lm}(\mathbf{R},T;\mathbf{k},\omega)d^3k d\omega. \tag{28}$$

Our next goal is to express $\boldsymbol{\mathcal{E}}$ by the correlation tensor $\tilde{v}_{ij}^{(0)}$. For this purpose we subject the differential equations for $u_i^{(1)}$, $b_i^{(0)}$, and $b_i^{(1)}$ given by (17) to a Fourier transformation, which results in algebraic equations for $\hat{u}_i^{(1)}$, $\hat{b}_i^{(0)}$, and $\hat{b}_i^{(1)}$. In addition we apply the projection operator $P_{ij}(\mathbf{k})=\delta_{ij} - k_ik_j/k^2$ on that for $\hat{u}_i^{(1)}$. In this way we obtain

$$\hat{u}_i^{(1)} = N(\mathbf{k},\omega)\left[-U_{ij}\hat{u}_j^{(0)} + U_{jk}\left(k_j\frac{\partial\hat{u}_i^{(0)}}{\partial k_k} + 2\frac{k_ik_j}{k^2}\hat{u}_k^{(0)}\right) + \Omega_{ij}\hat{u}_j^{(0)}\right],$$

$$\hat{b}_i^{(0)} = E(\mathbf{k},\omega)\left(i(\mathbf{k}\cdot\mathbf{B})\hat{u}_i^{(0)} - B_{ij}\hat{u}_j^{(0)} - B_{jk}k_j\frac{\partial\hat{u}_i^{(0)}}{\partial k_k}\right),$$

$$\hat{b}_i^{(1)} = E(\mathbf{k},\omega)\left(i(\mathbf{k}\cdot\mathbf{B})\hat{u}_i^{(1)} - B_{ij}\hat{u}_j^{(1)} - B_{jk}k_j\frac{\partial\hat{u}_i^{(1)}}{\partial k_k} + U_{ij}\hat{b}_j^{(0)} + U_{jk}k_j\frac{\partial\hat{b}_i^{(0)}}{\partial k_k}\right),$$

$$\hat{u}_i^{(0)}k_i = \hat{u}_i^{(1)}k_i = \hat{b}_i^{(0)}k_i = \hat{b}_i^{(1)}k_i = 0 \tag{29}$$

with the abbreviations N, E, and Ω_{ij} defined by

$$N(\mathbf{k},\omega) = \frac{1}{\nu k^2 - i\omega}, \quad E(\mathbf{k},\omega) = \frac{1}{\eta k^2 - i\omega},$$

$$\Omega_{ij}(\mathbf{k}) = 2\epsilon_{ijk}\frac{(\mathbf{k}\cdot\boldsymbol{\Omega})}{k^2}k_k. \tag{30}$$

D. Calculation of $\boldsymbol{\mathcal{E}}^{(0)}$

We consider now $\boldsymbol{\mathcal{E}}$ and the corresponding quantities like a_{ij} and b_{ijk} at $\mathbf{R}=\mathbf{0}$ and $T=0$. If we drop the arguments $\mathbf{R}$ and T we always refer to $\mathbf{R}=\mathbf{0}$ and $T=0$. As already mentioned we restrict ourselves on an approximation in which $\boldsymbol{\mathcal{E}}$ consists only of the terms $\boldsymbol{\mathcal{E}}^{(0)}$ and $\boldsymbol{\mathcal{E}}^{(1)}$ in the expansion (16).

Let us start with $\boldsymbol{\mathcal{E}}^{(0)}$. Clearly $\boldsymbol{\mathcal{E}}^{(0)}$ and the corresponding contributions $a_{ij}^{(0)}$ and $b_{ijk}^{(0)}$ to a_{ij} and b_{ijk} are independent of $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$. In view of $\boldsymbol{\mathcal{E}}^{(0)}$ we consider first the contribution $\chi_{jk}^{(0)}$ to χ_{jk}. By reasons which will become clear soon we consider for a moment $\chi_{jk}^{(0)}(\mathbf{R},T)$ with arbitrary $\mathbf{R}$ and T and will go only later to the limit $\mathbf{R}\to\mathbf{0}$ and set $T=0$. We introduce the notation

$$[f(\mathbf{k},\omega)]_+ = f(\mathbf{k}+\mathbf{K}/2,\omega+\Omega/2),$$

$$[f(\mathbf{k},\omega)]_- = f(-\mathbf{k}+\mathbf{K}/2,-\omega+\Omega/2), \tag{31}$$

where f means an arbitrary function. Then we have

$$\chi_{lm}^{(0)}(\mathbf{R},T) = \int\int\int\int \Big\langle [\hat{u}_l^{(0)}]_+ \times \left[iB_j E k_j \hat{u}_m^{(0)} - B_{jk}\left(E\delta_{jm}\hat{u}_k^{(0)} - Ek_j \frac{\partial \hat{u}_m^{(0)}}{\partial k_k}\right)\right]_- \Big\rangle \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega. \quad (32)$$

For the sake of simplicity we have dropped the arguments $\mathbf{k}$ and ω of $\hat{u}_i^{(0)}$ and E.

For the evaluation of this and similar integrals two relations are of particular interest. To explain them we note first that

$$\left(\frac{\partial f(\mathbf{k},\omega)}{\partial k_i}\right)_+ = \left(\frac{1}{2}\frac{\partial}{\partial k_i} + \frac{\partial}{\partial K_i}\right)[f(\mathbf{k},\omega)]_+,$$

$$\left(\frac{\partial f(\mathbf{k},\omega)}{\partial k_i}\right)_- = -\left(\frac{1}{2}\frac{\partial}{\partial k_i} - \frac{\partial}{\partial K_i}\right)[f(\mathbf{k},\omega)]_- \quad (33)$$

and

$$\left(\frac{1}{2}\frac{\partial}{\partial k_i} - \frac{\partial}{\partial K_i}\right)[f(\mathbf{k},\omega)]_+ = 0,$$

$$\left(\frac{1}{2}\frac{\partial}{\partial k_i} + \frac{\partial}{\partial K_i}\right)[f(\mathbf{k},\omega)]_- = 0. \quad (34)$$

On this basis we find with the help of integrations by parts

$$\int\int\int\int [F(\mathbf{k},\omega)]_+[G(\mathbf{k},\omega)]_-\left(\frac{\partial H(\mathbf{k},\omega)}{\partial k_i}\right)_- \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega = -\int\int\int\int [F(\mathbf{k},\omega)]_+\left(\frac{\partial G(\mathbf{k},\omega)}{\partial k_i}\right)_-[H(\mathbf{k},\omega)]_- \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega + O(\mathbf{R}) \quad (35)$$

and an analogous relation with $[\cdots]_+$ exchanged by $[\cdots]_-$ and vice versa.

Starting from (32) and using (35) we find

$$\chi_{lm}^{(0)}(\mathbf{R},T) = \int\int\int\int \Big\{ iB_j [Ek_j]_- \langle [\hat{u}_l^{(0)}]_+ [\hat{u}_m^{(0)}]_-\rangle - B_{jk}\Big[[E]_-\delta_{jm}\langle [\hat{u}_l^{(0)}]_+[\hat{u}_k^{(0)}]_-\rangle - \left(\frac{\partial}{\partial k_k}(Ek_j)\right)_- \langle [\hat{u}_l^{(0)}]_+[\hat{u}_m^{(0)}]_-\rangle \Big]\Big\} \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega + O(\mathbf{R}). \quad (36)$$

We conclude then

$$a_{ij}^{(0)}(\mathbf{R},T) = i\epsilon_{ilm}\int\int\int\int [Ek_j]_-\langle [\hat{u}_l^{(0)}]_+[\hat{u}_m^{(0)}]_-\rangle \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega + O(\mathbf{R}) \quad (37)$$

and

$$b_{ijk}^{(0)}(\mathbf{R},T) = -\epsilon_{ilm}\int\int\int\int \Big[[E]_-\delta_{jm}\langle [\hat{u}_l^{(0)}]_+[\hat{u}_k^{(0)}]_-\rangle - \left(\frac{\partial}{\partial k_k}(Ek_j)\right)_- \langle [\hat{u}_l^{(0)}]_+[\hat{u}_m^{(0)}]_-\rangle \Big] \times \exp\{i[(\mathbf{KR}) - \Omega T]\} d^3K d\Omega d^3k d\omega + O(\mathbf{R}). \quad (38)$$

We assume that all mean quantities vary only weakly with $\mathbf{R}$ and not with T. In that sense we expand $[Ek_j]_-$ in (37) in a series with respect to $\mathbf{K}$ but neglect all terms of higher than first order in $\mathbf{K}$, and set $\Omega=0$. The first-order terms have factors K_i under the integrals, and these correspond to the application of the operator $-i\nabla_i$ to the function defined by these integrals without K_i. Proceeding now to the limit $\mathbf{R}\to\mathbf{0}$ and $T=0$, writing simply $a_{ij}^{(0)}$ instead of $a_{ij}^{(0)}(\mathbf{0},0)$ and remembering the definition of $\tilde{v}_{ij}^{(0)}(\mathbf{R},T,\mathbf{k},\omega)$, we find

$$a_{ij}^{(0)} = -\epsilon_{ilm}\int\int \left[E^*\left(ik_j - \frac{1}{2}\nabla_j\right) - E^{*\prime}\frac{k_j}{2k}(\mathbf{k}\cdot\nabla)\right]\tilde{v}_{lm}^{(0)} d^3k d\omega. \quad (39)$$

Here E^* stand for the complex conjugate of $E(\mathbf{k},\omega)$, which is equal to $E(\mathbf{k},-\omega)$. Note that E^* depends only via k on $\mathbf{k}$. For this type of functions we use the notation $f'=\partial f/\partial k$. Furthermore $\tilde{v}_{ij}^{(0)}$ and $\nabla_k\tilde{v}_{ij}^{(0)}$ stands for $\tilde{v}_{ij}^{(0)}(\mathbf{0},0,\mathbf{k},\omega)$ and $[\nabla_k\tilde{v}_{ij}^{(0)}(\mathbf{R},0,\mathbf{k},\omega)]_{\mathbf{R}=\mathbf{0}}$, respectively.

Starting from (38) for $b_{ijk}^{(0)}(\mathbf{R},T)$ we proceed analogously. Since, however, b_{ijk} is connected with the derivatives of $\overline{\mathbf{B}}$ we replace $[E]_-$ and $[\partial(Ek_j)/\partial k_k]_-$ simply by their values at $\mathbf{K}=\mathbf{0}$ and $\Omega=0$, that is, ignore any derivatives of $\tilde{v}_{ij}^{(0)}$. So we arrive at

$$b_{ijk}^{(0)} = \int\int \left(\epsilon_{ijl}E^*\tilde{v}_{lk}^{(0)} + \epsilon_{ilm}E^{*\prime}\frac{k_jk_k}{k}\tilde{v}_{lm}^{(0)}\right) d^3k d\omega. \quad (40)$$

We have dropped contributions to $b_{ijk}^{(0)}$ proportional to δ_{jk}, which because of $\nabla\cdot\overline{\mathbf{B}}=0$ do not contribute to $\boldsymbol{\mathcal{E}}$.

The results (39) and (40) agree with earlier ones, e.g., those given in Ref. [2].

E. Calculation of $\boldsymbol{\mathcal{E}}^{(1)}$

Let us now consider $\boldsymbol{\mathcal{E}}^{(1)}$ and the corresponding contributions $a_{ij}^{(1)}$ and $b_{ijk}^{(1)}$ to a_{ij} and b_{ijk}. $\boldsymbol{\mathcal{E}}^{(1)}$ is a sum of three terms, the first one linear and homogeneous in $\boldsymbol{\Omega}$ and the second and third ones linear and homogeneous in $\mathbf{W}$ or $\mathbf{D}$, respectively. Likewise $a_{ij}^{(1)}$ and $b_{ijk}^{(1)}$ are sums of three terms, which are again linear and homogeneous in $\boldsymbol{\Omega}$, $\mathbf{W}$, and $\mathbf{D}$. We denote the corresponding contributions to $a_{ij}^{(1)}$ and $b_{ijk}^{(1)}$ by $a_{ij}^{(\Omega)}$, $a_{ij}^{(W)}$, $a_{ij}^{(D)}$, $b_{ijk}^{(\Omega)}$, $b_{ijk}^{(W)}$, and $b_{ijk}^{(D)}$.

We may calculate the latter quantities in the same way as we did it with $a_{ij}^{(0)}$ and $b_{ijk}^{(0)}$. Unfortunately the results are rather bulky. Some simplification is possible if we split $\tilde{v}_{ij}^{(0)}$ into its symmetric and antisymmetric part,

$$\tilde{v}_{ij}^{(0)} = \tilde{v}_{ij}^{(s)} + \tilde{v}_{ij}^{(a)}, \quad \tilde{v}_{ij}^{(s)} = \tilde{v}_{ji}^{(s)}, \quad \tilde{v}_{ij}^{(a)} = -\tilde{v}_{ji}^{(a)} \tag{41}$$

and assume that the symmetric part is even and the antisymmetric one is odd in $\mathbf{k}$,

$$\tilde{v}_{ij}^{(s)}(\mathbf{k},\omega) = \tilde{v}_{ij}^{(s)}(-\mathbf{k},\omega),$$

$$\tilde{v}_{ij}^{(a)}(\mathbf{k},\omega) = -\tilde{v}_{ij}^{(a)}(-\mathbf{k},\omega). \tag{42}$$

This assumption is true for any homogeneous turbulence and also for the form of inhomogeneous turbulence which we will consider later.

The results of the calculations for $a_{ij}^{(\Omega)}$ and $b_{ijk}^{(\Omega)}$ read

$$\begin{aligned} a_{ij}^{(\Omega)} = \int\int \Bigg\{ & E^*(N-N^*)\frac{(\mathbf{k}\cdot\mathbf{\Omega})}{k^2}k_i\nabla_j\tilde{v}_{ll}^{(s)} \\ & - E^*(N+N^*)\left[\frac{(\mathbf{k}\cdot\mathbf{\Omega})}{k^2}\left(k_j\nabla_i - 2\frac{k_ik_j}{k^2}(\mathbf{k}\cdot\mathbf{\nabla})\right)\tilde{v}_{ll}^{(s)} \right. \\ & \left. + \frac{k_ik_j}{k^2}(\mathbf{\Omega}\cdot\mathbf{\nabla})\tilde{v}_{ll}^{(s)} - 2\frac{k_j(\mathbf{k}\cdot\mathbf{\Omega})}{k^2}\nabla_l\tilde{v}_{li}^{(s)}\right] \\ & + [E^{*\prime}(N-N^*) - E^*(N'+N^{*\prime})] \\ & \times\frac{k_ik_j(\mathbf{k}\cdot\mathbf{\Omega})}{k^3}(\mathbf{k}\cdot\mathbf{\nabla})\tilde{v}_{ll}^{(s)}\Bigg\}d^3k\,d\omega, \end{aligned} \tag{43}$$

$$\begin{aligned} b_{ijk}^{(\Omega)} = -2\int\int \frac{(\mathbf{k}\cdot\mathbf{\Omega})}{k^2}\bigg(& E^*(N+N^*)(k_i\tilde{v}_{jk}^{(s)} - k_j\tilde{v}_{ik}^{(s)}) \\ & + E^*N^*\delta_{ik}k_j\tilde{v}_{ll}^{(s)} - E^{*\prime}(N-N^*)\frac{k_ik_jk_k}{k}\tilde{v}_{ll}^{(s)}\bigg)d^3k\,d\omega. \end{aligned} \tag{44}$$

Again E^* stands for the complex conjugate of $E(\mathbf{k},\omega)$, that is for $E(\mathbf{k},-\omega)$. Likewise N means $N(\mathbf{k},\omega)$ and N^* its complex conjugate, that is $N(\mathbf{k},-\omega)$. As before $\tilde{v}_{ij}^{(0)}$ and $\nabla_m\tilde{v}_{ij}^{(0)}$ mean $\tilde{v}_{ij}^{(0)}(\mathbf{0},0,\mathbf{k},\omega)$ and $[\nabla_m\tilde{v}_{ij}^{(0)}(\mathbf{R},0,\mathbf{k},\omega)]_{\mathbf{R}=\mathbf{0}}$, respectively. As in the case of $b_{ijk}^{(0)}$ contributions to $b_{ijk}^{(\Omega)}$ with δ_{jk} have been dropped.

The corresponding results for $a_{ij}^{(W)}$, $a_{ij}^{(D)}$, $b_{ijk}^{(W)}$, and $b_{ijk}^{(D)}$ are given in Appendix A.

F. Results for $\mathcal{E}$ with a specific velocity correlation tensor

We now specify the correlation tensor $\tilde{v}_{ij}^{(0)}(\mathbf{R},T,\mathbf{k},\omega)$ so that it corresponds to an inhomogeneous turbulence deviating from a homogeneous isotropic mirror symmetric and statistically steady one only by a gradient of the turbulence intensity. In that sense we set

$$\tilde{v}_{ij}^{(0)}(\mathbf{R},T,\mathbf{k},\omega) = \frac{1}{2}\left(P_{ij}(\mathbf{k}) + \frac{i}{2k^2}(k_i\nabla_j - k_j\nabla_i)\right)W(\mathbf{R},T,k,\omega), \tag{45}$$

where again $P_{ij}(\mathbf{k})=(\delta_{ij}-k_ik_j/k^2)$. Here $W(\mathbf{R},T,k,\omega)$ is the Fourier transform of $\langle\mathbf{u}(\mathbf{R}+\mathbf{r}/2,T+t/2)\mathbf{u}(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle$ with respect to $\mathbf{r}$ and t,

$$\begin{aligned} \int\int W(\mathbf{R},T,k,\omega)\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]d^3k\,d\omega \\ = \langle\mathbf{u}(\mathbf{R}+\mathbf{r}/2,T+t/2)\cdot\mathbf{u}(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle; \end{aligned} \tag{46}$$

see also Ref. [10]. Note that (45) satisfies both (27) and (42). Anticipating that we will later specify $W(\mathbf{R},T,k,\omega)$ as a product of a factor $\langle u^{(0)2}\rangle$ depending on $\mathbf{R}$ and T and a factor depending on k and ω only we set

$$\nabla W(\mathbf{R},T,k,\omega) = gW(\mathbf{R},T,k,\omega) \tag{47}$$

and interpret $\mathbf{g}$ as $\mathbf{\nabla}\langle\mathbf{u}^{(0)2}\rangle/\langle\mathbf{u}^{(0)2}\rangle$.

We now specify the results for $a_{ij}^{(0)}$, $a_{ij}^{(\Omega)}, \ldots, b_{ijk}^{(D)}$ given by (39), (40), (43), (44), and (A1)–(A4) with the ansatz (45) for $\tilde{v}_{ij}^{(0)}$. We further use the relations

$$\int k_ik_jf(k)d^3k = \frac{1}{3}\delta_{ij}\int k^2f(k)d^3k,$$

$$\int k_ik_jk_kk_lf(k)d^3k = \frac{1}{15}(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\int k^4f(k)d^3k, \tag{48}$$

which apply for all functions f depending on $\mathbf{k}$ only via k. The integrals are over all $\mathbf{k}$.

In this way we find results for the coefficients $\gamma^{(0)}$, $\beta^{(0)}$, $\alpha_1^{(\Omega)}, \ldots, \kappa^{(D)}$, say generally f, in the form

$$f = 4\pi\int_{k=0}^{\infty}\int_{\omega=-\infty}^{\infty}\tilde{f}(k,\omega)W(k,\omega)k^2dk\,d\omega. \tag{49}$$

As for $\gamma^{(0)}$, $\beta^{(0)}$, $\alpha_1^{(\Omega)}, \ldots, \kappa^{(\Omega)}$ the $\tilde{f}$ are given by

$$2\tilde{\gamma}^{(0)} = \tilde{\beta}^{(0)} = \frac{1}{3}\frac{\eta k^2}{(\eta k^2)^2+\omega^2},$$

$$\tilde{\alpha}_1^{(\Omega)} = \frac{4}{15}\left(\frac{\eta\nu k^4[(\nu k^2)^2+3\omega^2]}{[(\eta k^2)^2+\omega^2][(\nu k^2)^2+\omega^2]^2} + \frac{2(\eta k^2)^2\omega^2}{[(\eta k^2)^2+\omega^2]^2[(\nu k^2)^2+\omega^2]}\right),$$

$$\tilde{\alpha}_2^{(\Omega)} = -\frac{1}{15}\left(\frac{2\eta\nu k^4(3(\nu k^2)^2-\omega^2)}{[(\eta k^2)^2+\omega^2][(\nu k^2)^2+\omega^2]^2} - \frac{[3(\eta k^2)^2-5\omega^2]\omega^2}{[(\eta k^2)^2+\omega^2]^2[(\nu k^2)^2+\omega^2]}\right),$$

$$\tilde{\gamma}^{(\Omega)} = \tilde{\delta}^{(\Omega)} = -\frac{1}{3}\frac{\omega^2}{[(\eta k^2)^2+\omega^2][(\nu k^2)^2+\omega^2]},$$

$$\tilde{\kappa}^{(\Omega)} = \frac{2}{15}\frac{(11(\eta k^2)^2 - 5\omega^2)\omega^2}{[(\eta k^2)^2 + \omega^2]^2[(\nu k^2)^2 + \omega^2]}. \quad (50)$$

The corresponding results for $\alpha_1^{(W)}, \alpha_1^{(W)}, \ldots, \kappa^{(D)}$ are given in Appendix B. Note that not only $\gamma^{(0)}$ and $\beta^{(0)}$ are independent of ν but also $\delta^{(W)}$. Whereas this independence is quite natural for $\gamma^{(0)}$ and $\beta^{(0)}$, it results from an accidental compensation of contributions in the case of $\delta^{(W)}$.

G. Specific results

Let us now calculate the coefficients $\gamma^{(0)}$, $\beta^{(0)}$, $\alpha_1^{(\Omega)}$, $\alpha_2^{(\Omega)}, \ldots, \kappa^{(D)}$ according to (49), (50), and (B1) with a specific ansatz for $W(\mathbf{R},T;\mathbf{k},\omega)$, that is for $\langle \mathbf{u}(\mathbf{R}+\mathbf{r}/2,T+t/2)\mathbf{u}(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle$. We set

$$\langle u(\mathbf{R}+\mathbf{r}/2,T+t/2)\mathbf{u}(\mathbf{R}-\mathbf{r}/2,T-t/2)\rangle = \overline{u^2}(\mathbf{R},T)\exp(-r^2/2\lambda_c^2 - t/|\tau_c|). \quad (51)$$

Simplifying the notation we have written $\overline{u^2}$ instead of $\langle u^{(0)2}\rangle$, that is, $\overline{u^2}$ describes the turbulence intensity in the limit of vanishing Coriolis force and mean velocity gradient. Further λ_c and τ_c are correlation length and time in this limit. We refrain here from considering λ_c and τ_c as functions of k and ω. Because of (46) relation (51) is equivalent to

$$W = \overline{u^2}(\mathbf{R},T)\frac{2\lambda_c^3\tau_c}{3(2\pi)^{5/2}}\frac{(k\lambda_c)^2\exp[-(k\lambda_c)^2/2]}{1+(\omega\tau_c)^2}. \quad (52)$$

In what follows we use the dimensionless parameters

$$q = \lambda_c^2/\eta\tau_c, \quad p = \lambda_c^2/\nu\tau_c, \quad P_m = \nu/\eta. \quad (53)$$

The quantity q is the ratio of the magnetic diffusion time λ_c^2/η to the correlation time τ_c. We speak simply of low-conductivity limit if $q \to 0$, and of high-conductivity limit if $q \to \infty$, knowing that these limits can also be reached with any finite η but $\tau_c \to \infty$ or $\tau_c \to 0$, respectively. Likewise p is the ratio of the hydrodynamic decay time λ_c^2/ν to the correlation time τ_c, and $p \to 0$ and $p \to \infty$ are denoted as the high and low viscosity limits, respectively. Pm is the magnetic Prandtl number of the fluid, and it holds $P_m = q/p$. Furthermore we introduce the magnetic Reynolds number Rm, the hydrodynamic Reynolds number Re, and the Strouhal number St by

$$\mathrm{Rm} = \frac{u_c\lambda_c}{\eta}, \quad \mathrm{Re} = \frac{u_c\lambda_c}{\nu}, \quad \mathrm{St} = \frac{u_c\tau_c}{\lambda_c}, \quad (54)$$

where $u_c = \sqrt{\overline{u^2}}$. For a realistic turbulence St is close to unity. Then q and p are close to Rm and Re, respectively.

We return now to the representation (12) for $\boldsymbol{\mathcal{E}}$, again with $\mathbf{g} = \nabla\overline{u^2}/\overline{u^2}$. We give our results for the coefficients in this representation first in a form suitable for application to the dynamo experiment mentioned above, where q is at least not large compared to unity. This form reads

$$\alpha_1^{(\Omega)} = (4/45)\mathrm{Rm}^2\,\lambda_c^2\alpha_1^{o(\Omega)}(P_m,q),$$

$$\alpha_2^{(\Omega)} = -(2/15)\mathrm{Rm}^2\,\lambda_c^2\alpha_2^{o(\Omega)}(P_m,q),$$

$$\alpha_1^{(W)} = (19/360)\mathrm{Rm}^2\,\lambda_c^2\alpha_1^{o(W)}(P_m,q),$$

$$\alpha_2^{(W)} = -(7/720)\mathrm{Rm}^2\,\lambda_c^2\alpha_2^{o(W)}(P_m,q),$$

$$\alpha^{(D)} = -(7/120)\mathrm{Rm}^2\,\lambda_c^2\alpha^{o(D)}(P_m,q),$$

$$\gamma^{(0)} = \tfrac{1}{18}\mathrm{Rm}^2\,\eta\gamma^{o(0)}(q),$$

$$\gamma^{(\Omega)} = -(\sqrt{\pi}/36\sqrt{2})\mathrm{Rm}^2\,\lambda_c^2\sqrt{q}\gamma^{o(\Omega)}(P_m,q),$$

$$\gamma^{(W)} = -(1/144)\mathrm{Rm}^2\,\lambda_c^2\gamma^{o(W)}(P_m,q),$$

$$\gamma^{(D)} = -(13/120)\mathrm{Rm}^2\,\lambda_c^2\gamma^{o(D)}(P_m,q), \quad (55)$$

$$\beta^{(0)} = (1/9)\mathrm{Rm}^2\,\eta\beta^{o(0)}(q),$$

$$\beta^{(D)} = (7/90)\mathrm{Rm}^2\,\lambda_c^2\beta^{o(D)}(P_m,q),$$

$$\delta^{(\Omega)} = -(\sqrt{\pi}/36\sqrt{2})\mathrm{Rm}^2\,\lambda_c^2\sqrt{q}\delta^{o(\Omega)}(P_m,q),$$

$$\delta^{(W)} = (1/36)\mathrm{Rm}^2\,\lambda_c^2\delta^{o(W)}(q),$$

$$\kappa^{(\Omega)} = (\sqrt{\pi}/18\sqrt{2})\mathrm{Rm}^2\,\lambda_c^2\sqrt{q}\kappa^{o(\Omega)}(P_m,q),$$

$$\kappa^{(W)} = -(1/90)\mathrm{Rm}^2\,\lambda_c^2\kappa^{o(W)}(P_m,q),$$

$$\kappa^{(D)} = (13/90)\mathrm{Rm}^2\,\lambda_c^2\kappa^{o(D)}(P_m,q). \quad (56)$$

The numerical factors are chosen such that the functions $\alpha_1^{o(\Omega)}, \alpha_1^{o(\Omega)}, \ldots, \kappa^{o(\Omega)}$ with $P_m = 1$ approach unity in the low-conductivity limit $q \to 0$. According to (49) and (50) we have $\gamma^{o(0)} = \beta^{o(0)}$ and $\gamma^{o(\Omega)} = \delta^{o(\Omega)}$. Figure 1 shows the dependence of the functions $\alpha_1^{o(\Omega)}, \alpha_1^{o(\Omega)}, \ldots, \kappa^{o(\Omega)}$ on P_m and q.

In astrophysical applications the high-conductivity limit $q \to \infty$ is of particular interest. Then a modified representation of these results seems appropriate,

$$\alpha_1^{(\Omega)} = (4/45)\overline{u^2}\tau_c^2\alpha_1^{\infty(\Omega)}(p,q),$$

$$\alpha_2^{(\Omega)} = -(1/90)(22-5\xi)\overline{u^2}\tau_c^2\alpha_2^{\infty(\Omega)}(p,q),$$

$$\alpha_1^{(W)} = (1/72)(\xi-1)\overline{u^2}\tau_c^2\alpha_1^{\infty(W)}(p,q),$$

$$\alpha_2^{(W)} = -(1/144)(11+\xi)\overline{u^2}\tau_c^2\alpha_2^{\infty(W)}(p,q),$$

$$\alpha^{(D)} = -(1/360)(29-5\xi)\overline{u^2}\tau_c^2\alpha^{\infty(D)}(p,q),$$

$$\gamma^{(0)} = (1/6)\overline{u^2}\tau_c\gamma^{\infty(0)}(q),$$

$$\gamma^{(\Omega)} = -(1/18)(2-\xi)\overline{u^2}\tau_c^2\gamma^{\infty(\Omega)}(p,q),$$

$$\gamma^{(W)} = -(1/144)(13+\xi))\overline{u^2}\tau_c^2\gamma^{\infty(W)}(p,q),$$

$$\gamma^{(D)} = -(1/72)(7-\xi)\overline{u^2}\tau_c^2\gamma^{\infty(D)}(p,q), \quad (57)$$

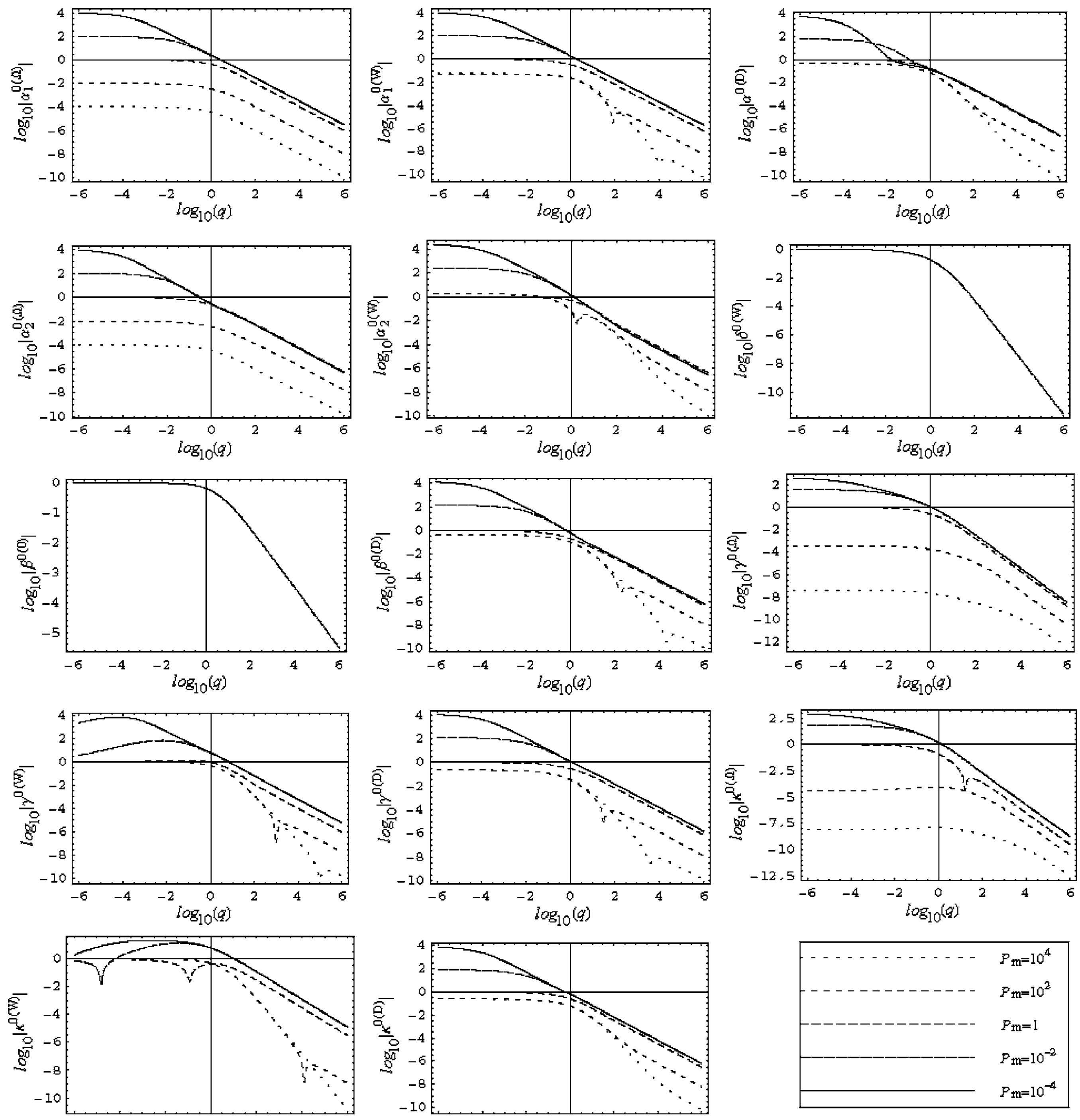

FIG. 1. The dependence of the coefficients $\alpha_1^{o(\Omega)}$, $\alpha_1^{o(\Omega)}, \cdots, \kappa^{o(\Omega)}$ on P_m and q. Note that $\gamma^{o(0)}$ coincides with $\beta^{o(0)}$, and $\delta^{o(\Omega)}$, with $\gamma^{o(\Omega)}$. The different line styles correspond to different values of P_m, see the last frame. For all P_m these coefficients are positive as long as q is small. In some cases the signs change as q grows. This is indicated by tips of the curves.

$$\beta^{(0)} = (1/3)\overline{u^2}\tau_c \beta^{\infty(0)}(q),$$

$$\beta^{(D)} = -(7/90)\overline{u^2}\tau_c^2 \beta^{\infty(D)}(p,q),$$

$$\delta^{(\Omega)} = -(1/18)(2-\xi)\overline{u^2}\tau_c^2 \delta^{\infty(\Omega)}(p,q),$$

$$\delta^{(W)} = (1/12)\overline{u^2}\tau_c^2 \delta^{\infty(W)}(q),$$

$$\kappa^{(\Omega)} = -(1/9)(2-\xi)\overline{u^2}\tau_c^2 \kappa^{\infty(\Omega)}(p,q),$$

$$\kappa^{(W)} = -(1/6)\overline{u^2}\tau_c^2 \kappa^{\infty(W)}(p,q),$$

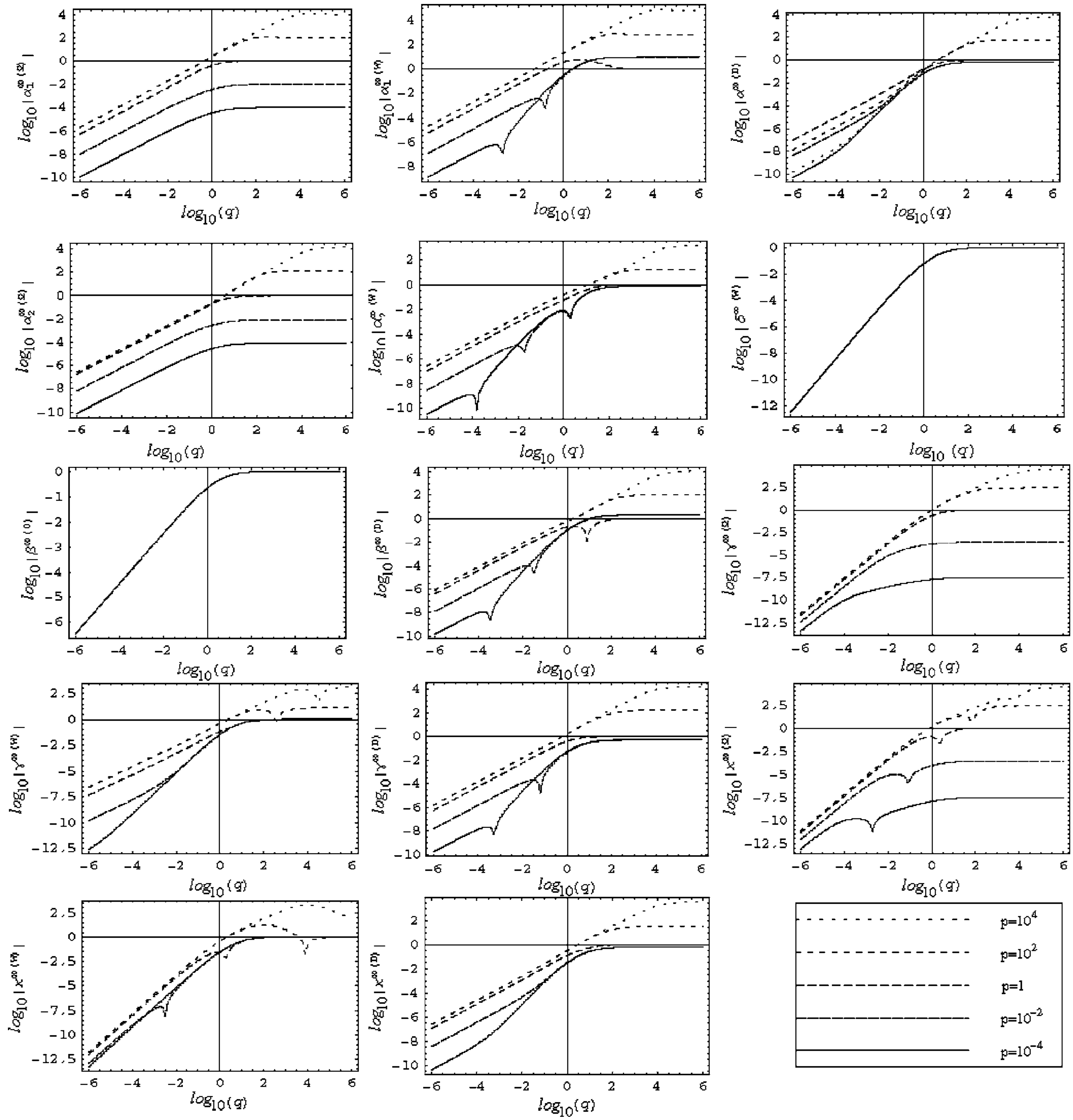

FIG. 2. The dependence of the coefficients $\alpha_1^{\infty(\Omega)}, \alpha_1^{\infty(\Omega)}, \ldots, \kappa^{\infty(\Omega)}$ on p and q. Note that $\gamma^{\infty(0)}$ coincides with $\beta^{\infty(0)}$, and $\delta^{\infty(\Omega)}$ with $\gamma^{\infty(\Omega)}$. The explanations given with Fig. 1 apply analogously but for all p these coefficients are positive as long as q is large.

$$\kappa^{(D)} = (23/90)\overline{u^2}\tau_c^2\kappa^{\infty(D)}(p,q), \tag{58}$$

where $\xi = \sqrt{2e}[\sqrt{\pi} - 2\int_0^{\sqrt{2}} \exp(-t^2)dt] \approx 1.31$. The functions $\alpha_1^{\infty(\Omega)}, \alpha_1^{\infty(\Omega)}, \ldots, \kappa^{\infty(\Omega)}$ are defined such that their values at $p=1$ approach unity as $q \to \infty$. Note that $\overline{u^2}\tau_c^2 = \mathrm{St}^2\lambda_c^2$. According to (49) and (50) we have now $\gamma^{\infty(0)} = \beta^{\infty(0)}$ and $\gamma^{\infty(\Omega)} = \delta^{\infty(\Omega)}$. The functions $\alpha_1^{\infty(\Omega)}, \alpha_1^{\infty(\Omega)}, \ldots, \kappa^{\infty(\Omega)}$ are shown in Fig. 2.

V. DISCUSSION

A. Assumptions and approximations

Our results have been gained with some assumptions and approximations. As usual it has been generally assumed that electromotive force $\boldsymbol{\mathcal{E}}$ depends in the linear and homogeneous form (4) on $\overline{\mathbf{B}}$. The only additional assumption introduced in Sec. III, just for the sake of simplicity, is the linearity of $\boldsymbol{\mathcal{E}}$ in the angular velocity $\boldsymbol{\Omega}$ and the gradient $\nabla\overline{\mathbf{U}}$ of the mean velocity, that is, some smallness of the Coriolis force and the shear in the mean motion. In Sec. IV, however, some kind of second-order correlation approximation (SOCA) has been introduced. As long as only results are considered which are independent of $\mathbf{\Omega}$ and $\nabla\bar{\mathbf{U}}$, our procedure corresponds to the traditional second-order correlation approximation; see, e.g., Ref. [2]. In the low-conductivity limit, $q \to 0$, a sufficient condition for the validity of these results is $\mathrm{Rm} \ll 1$. In the high-conductivity limit, $q \to \infty$, the corresponding condition reads $\mathrm{St} \ll 1$. If nonzero $\mathbf{\Omega}$ and $\nabla\bar{\mathbf{U}}$ are taken into account, additional conditions expressing the smallness of their effects on the fluid motion must be satisfied. These conditions are roughly described in Sec. IV A.

B. Former results

There is a series of former results for situations covered by our assumptions. We refer in particular to those in the early works by Steenbeeck *et al.* [28], Krause *et al.* [1], Rädler [29], further to those by Vainshtein *et al.* [7], Rüdiger *et al.* [8], and Kichatinov *et al.* [9]. As far as these results are given in a form that allows a detailed comparison our results are in satisfying agreement with most of them. We note that in the calculations by Steenbeck *et al.* [28], which revealed the α effect, due to an incorrect assumption on the velocity correlation tensor, the $\mathbf{\Omega} \times \mathbf{J}$ effect does not occur. The latter was found only later [29].

We also point out the recent papers by Rädler, Kleeorin, and Rogachevskii [10] (referred to as RKR03 in the following) and by Rogachevskii and Kleeorin [14] (referred to as RK03). In both papers an approach is used, which is aimed to go beyond the second-order correlation approximation by taking into account higher-order correlations of $\mathbf{u}$ and $\mathbf{b}$ at least in some crude way. It was suggested by the τ approximation of turbulence theory and is therefore called "τ approach" in the following. Unfortunately, there is no parameter range in which it completely reproduces the results for the mean-field coefficients obtained with the second-order correlation approximation; see Ref. [30]. Possibly the assumptions of the τ approach, which rely on a developed turbulence with high hydrodynamic and magnetic Reynolds numbers, Re and Rm, exclude the assumptions used in the second-order correlation approach. Nevertheless some of the findings of the τ approach are of interest for the following.

C. New findings concerning the α, γ, and β effects

It is well known that an inhomogeneous turbulence at a rotating body gives rise to an α effect. In this case the essential construction elements of the tensor $\boldsymbol{\alpha}$ are the vectors $\mathbf{g}$ and $\mathbf{\Omega}$ describing the gradient in the turbulence intensity and the Coriolis force. Our results show in agreement with those by RK03 that even in the absence of a Coriolis force the combination of inhomogeneous turbulence, that is nonzero $\mathbf{g}$, and a gradient of the mean velocity, $\nabla\bar{\mathbf{U}}$, leads to an α effect. This is perhaps less surprising if the gradient of the mean velocity corresponds to a rotation. Then the role of $\mathbf{\Omega}$ in the tensor $\boldsymbol{\alpha}$ is played by $\mathbf{W}$. It is however quite remarkable that, again in combination with inhomogeneous turbulence, also the symmetric part of the mean velocity gradient, $\mathbf{D}$, which corresponds to a deformation, leads to an α effect. This contribution to $\boldsymbol{\alpha}$ has however some peculiarity, in particular its trace is equal to zero; see also Ref. [21].

In all models of α^2 or $\alpha\omega$ dynamos considered so far the contributions to the α effect depending on the shear of the mean flow have been ignored. It remains to be investigated how they modify the behavior of such dynamos, in particular that of an $\alpha\omega$ dynamo in the case of very strong differential rotation.

It is also known that the γ effect, which describes a transport of mean magnetic flux and occurs primarily as a consequence of a gradient of the turbulence intensity, is modified by the Coriolis force, that is, the vector $\boldsymbol{\gamma}$ contains a part with $\mathbf{\Omega}$. Our results show in agreement with RK03 that γ possesses also contributions with both parts of the mean velocity gradient $\nabla\bar{\mathbf{U}}$, that is, with $\mathbf{W}$ and $\mathbf{D}$.

In mean-field electrodynamics instead of the molecular magnetic diffusivity η the mean-field diffusivity $\eta + \beta^{(0)}$ occurs. More generally spoken, the tensor $\boldsymbol{\beta}$ must be added to the isotropic molecular diffusivity tensor. It is clear from simple symmetry considerations and can also be seen in RKR03 and in RK03 that there are no contributions to $\boldsymbol{\beta}$ depending on $\mathbf{\Omega}$ or $\mathbf{W}$ as long as we restrict ourselves to linearity in these quantities. We have found however, again in agreement with RK03, that there is a contribution proportional to the symmetric part of the mean velocity gradient $\nabla\bar{\mathbf{U}}$, that is to $\mathbf{D}$. The mean-field diffusivity, and so the mean-field conductivity, becomes anisotropic as a consequence of the deforming mean motion described by $\mathbf{D}$.

Since $\beta^{(0)}$ is always positive it raises the threshold of a dynamo. Interestingly enough the mean-field diffusivity tensor need not to be positive definite, and the β effect may then well support a dynamo, see Ref. [21].

D. New findings concerning the δ and κ effects

Proceeding to the δ and κ effects we mention first that already in the case of a homogeneous turbulence at a rotating body, that is, subject to the Coriolis force, contributions to the mean electromotive force proportional to $\mathbf{\Omega} \times (\nabla \times \bar{\mathbf{B}})$ and to $\mathbf{\Omega} \circ (\nabla\bar{\mathbf{B}})^{(s)}$ proved to be possible. They usually occur simultaneously. As already mentioned the occurrence of the first one is often referred to as $\mathbf{\Omega} \times \mathbf{J}$ effect. We note that

$$\delta^{(\Omega)}\mathbf{\Omega} \times (\nabla \times \bar{\mathbf{B}}) + \kappa^{(\Omega)}\mathbf{\Omega} \circ (\nabla\bar{\mathbf{B}})^{(s)} = \zeta_1^{(\Omega)}(\mathbf{\Omega} \cdot \nabla)\bar{\mathbf{B}} + \zeta_2^{(\Omega)} \nabla (\Omega \cdot \bar{\mathbf{B}}), \tag{59}$$

where

$$\zeta_1^{(\Omega)} = -\delta^{(\Omega)} + \tfrac{1}{2}\kappa^{(\Omega)}, \quad \zeta_2^{(\Omega)} = \delta^{(\Omega)} + \tfrac{1}{2}\kappa^{(\Omega)}. \tag{60}$$

As long as $\zeta_2^{(\Omega)}$ is independent of position the last term on the right-hand side is without interest for the induction equation. Then the $\delta^{(\Omega)}$ and $\kappa^{(\Omega)}$ effects act, apart from the signs, in the same way. Interestingly enough, $\zeta_1^{(\Omega)}$ vanishes in both limits $q \to 0$ and $q \to \infty$. As long as the ansatz (51) is adopted and therefore (56) and (58) apply, this can easily be seen for $P_m = 1$ and $q \to 0$ from (56), and for $p = 1$ and $q \to \infty$ from

(58). A more general proof of the above statement on $\zeta_1^{(\Omega)}$ is given in Appendix C.

Let us have a look on the results of the τ approach for $\delta^{(\Omega)}$ and $\kappa^{(\Omega)}$ given in RKR03. It seems plausible to interpret them as results for $q \to \infty$. The quantity $\zeta_1^{(\Omega)}$ calculated from them is equal to zero if the correlation time τ_c is considered as a constant, but it deviates from zero as soon as its Fourier transform depends on k. This is in conflict with the general result explained in Appendix C.

We recall that the $\delta^{(\Omega)}$ effect, even in the absence of any α effect, but in combination with differential rotation, is capable of dynamo action, see Refs. [23–26,31] and RKR03. Dynamos of that kind are often labeled as $\mathbf{\Omega} \times \mathbf{J}$ dynamos. Strictly speaking, both the $\delta^{(\Omega)}$ and the $\kappa^{(\Omega)}$ effects may constitute this dynamo mechanism if only $\zeta_1^{(\Omega)}$ is nonzero. As a consequence of the differential rotation, also induction effects connected with $\mathbf{W}$ and $\mathbf{D}$ necessarily play some part in $\mathbf{\Omega} \times \mathbf{J}$ dynamos but have not been considered so far.

Our above results show that besides the $\mathbf{\Omega} \times \mathbf{J}$ effect also an analogous $\mathbf{W} \times \mathbf{J}$ effect exists, which occurs even in the absence of the Coriolis force. This effect and the related ones have already been considered by Urpin [11,12] and extensively studied in RK03. However, details of the results by Urpin seem to be incorrect, and those of RK03 do not agree with ours, which is a consequence of the fact that the τ approach was used instead of the second-order correlation approximation. Analogous to (59) we have

$$\delta^{(W)}\mathbf{W} \times (\boldsymbol{\nabla} \times \overline{\mathbf{B}}) + \kappa^{(W)}\mathbf{W} \circ (\boldsymbol{\nabla}\overline{\mathbf{B}})^{(s)} = \zeta_1^{(W)}(\mathbf{W} \cdot \boldsymbol{\nabla})\overline{\mathbf{B}} + \zeta_2^{(W)}\mathbf{W} \circ (\boldsymbol{\nabla}\overline{\mathbf{B}}), \tag{61}$$

where

$$\zeta_1^{(W)} = -\delta^{(W)} + \tfrac{1}{2}\kappa^{(W)}, \quad \zeta_2^{(W)} = \delta^{(W)} + \tfrac{1}{2}\kappa^{(W)}. \tag{62}$$

Here $\mathbf{W} \circ (\boldsymbol{\nabla}\overline{\mathbf{B}})$ is defined by $[W \circ (\boldsymbol{\nabla}\overline{\mathbf{B}})]_i = W_j \partial \overline{B}_j / \partial x_i$. For constant $\mathbf{W}$ it is again a gradient. If then in addition $\zeta_2^{(W)}$ is independent of position the $\delta^{(W)}$ and $\kappa^{(W)}$ effects act again in the same way. In contrast to $\zeta_1^{(\Omega)}$ the coefficient $\zeta_1^{(W)}$ takes in general nonzero values as $q \to 0$ or $q \to \infty$.

Different from the situation with the $\delta^{(\Omega)}$ and $\kappa^{(\Omega)}$ effects, the $\delta^{(W)}$ and $\kappa^{(W)}$ effects are accompanied by the $\beta^{(D)}$ and $\kappa^{(D)}$ effects. Apart from the case in which $\overline{\mathbf{U}}$ corresponds to a rigid-body rotation, together with $\mathbf{W}$ also $\mathbf{D}$ is unequal to zero so that the $\beta^{(D)}$ and $\kappa^{(D)}$ effects indeed occur. This makes the comparison between the effects working with $\mathbf{\Omega}$ and those working with $\mathbf{W}$ more complex. Note that in contrast to the signs of $\alpha_1^{(\Omega)}$ and $\alpha_1^{(W)}$, of $\alpha_2^{(\Omega)}$ and $\alpha_2^{(W)}$ and of $\gamma^{(\Omega)}$ and $\gamma^{(W)}$, those of $\delta^{(\Omega)}$ and $\delta^{(W)}$ differ; with $\kappa^{(\Omega)}$ and $\kappa^{(W)}$ the situation depends on q.

Analogously to the $\mathbf{\Omega} \times \mathbf{J}$ dynamo an $\mathbf{W} \times \mathbf{J}$ dynamo was proposed in RK03, working with the induction effects of turbulence discussed here, which are due to a mean shear, and the induction effect due to the shear alone. In a simple model in Cartesian geometry, using results for $\delta^{(W)}$, $\kappa^{(W)}$, $\beta^{(D)}$, and $\kappa^{(D)}$ obtained in the τ approach, indeed growing $\overline{\mathbf{B}}$ were found. Recently Rüdiger *et al.* [32] pointed out that this model does not work as a dynamo with $\delta^{(W)}$, $\kappa^{(W)}$, $\beta^{(D)}$, and $\kappa^{(D)}$ as found in the second-order correlation approximation. Our consideration in Appendix D confirms this finding. We stress that our negative conclusion applies only to a simple model of the $\mathbf{W} \times \mathbf{J}$ dynamo and to the range of validity of the second-order correlation approximation. It remains to be checked whether this applies to other models, too. In cylindrical or spherical geometry the $\mathbf{\Omega} \times \mathbf{J}$ and $\mathbf{W} \times \mathbf{J}$ effects occur always simultaneously. The question of a pure $\mathbf{W} \times \mathbf{J}$ dynamo does not appear.

ACKNOWLEDGMENTS

The authors thank N. Kleeorin, I. Rogachevskii, and M. Rheinhardt for helpful comments on the paper. One of the authors (R.S.) appreciates the financial support from Grant No. ISTC-2021 and the BRHE Program.

APPENDIX A: RELATIONS FOR $a_{ij}^{(W)}$, $a_{ij}^{(D)}$, $b_{ijk}^{(W)}$, AND $b_{ijk}^{(D)}$

Analogous to the results (43) and (44) for $a_{ij}^{(\Omega)}$ and $b_{ijk}^{(\Omega)}$ we find

$$\begin{aligned}
a_{ij}^{(W)} = {} & \frac{i}{2}\int\int [E^*(N+N^*)k_j W_l \tilde{v}_{li}^{(a)} + 2E^*N^* k_i W_l \tilde{v}_{lj}^{(a)} + E^{*2} k_j W_l \tilde{v}_{li}^{(a)}] d^3k\, d\omega + \frac{1}{4}\int\int \Bigg[E^* N \Bigg(-W_i \nabla_j \tilde{v}_{ll}^{(s)} + W_l \nabla_j \tilde{v}_{li}^{(s)} \\
& + 4\frac{k_j(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_l \tilde{v}_{li}^{(s)} + 2\frac{k_i(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_j \tilde{v}_{ll}^{(s)} - 2\frac{k_i k_j}{k^2}(\mathbf{W}\cdot\boldsymbol{\nabla})\tilde{v}_{ll}^{(s)} + 4\frac{k_i k_j(\mathbf{W}\cdot\mathbf{k})}{k^4}(\mathbf{k}\cdot\boldsymbol{\nabla})\tilde{v}_{ll}^{(s)} - 2\frac{k_j(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_i \tilde{v}_{ll}^{(s)} \Bigg) + E^* N^* \Bigg(W_i (\nabla_j \tilde{v}_{ll}^{(s)} \\
& + 2\nabla_l \tilde{v}_{lj}^{(s)}) - W_l \nabla_j \tilde{v}_{li}^{(s)} - 2\delta_{ij} W_l \nabla_n \tilde{v}_{ln}^{(s)} - 2\frac{k_i k_j}{k^2}(\mathbf{W}\cdot\boldsymbol{\nabla})\tilde{v}_{ll}^{(s)} + 4\frac{k_i k_j(\mathbf{W}\cdot\mathbf{k})}{k^4}(\mathbf{k}\cdot\boldsymbol{\nabla})\tilde{v}_{ll}^{(s)} - 2\frac{k_i(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_j \tilde{v}_{ll}^{(s)} + 4\frac{k_j(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_l \tilde{v}_{li}^{(s)} \\
& - 2\frac{k_j(\mathbf{W}\cdot\mathbf{k})}{k^2}\nabla_i \tilde{v}_{ll}^{(s)} \Bigg) - [E^{*\prime}N - E^* N' - (E^* N^*)'] \frac{k_j}{k}(\mathbf{k}\cdot\boldsymbol{\nabla}) \Bigg(W_i \tilde{v}_{ll}^{(s)} - W_l \tilde{v}_{li}^{(s)} - 2\frac{k_i(\mathbf{W}\cdot\mathbf{k})}{k^2}\tilde{v}_{ll}^{(s)} \Bigg) + \Bigg(E^{*2}\nabla_j + (E^{*2})' \frac{k_j}{k}(\mathbf{k}\cdot\boldsymbol{\nabla}) \Bigg) \\
& \times (W_i \tilde{v}_{ll}^{(s)} - W_l \tilde{v}_{li}^{(s)}) \Bigg] d^3k\, d\omega,
\end{aligned} \tag{A1}$$

$$a_{ij}^{(D)} = i\epsilon_{ilm}\int\int\Bigg[E^*(N+N^*)k_j\left(D_{mn}-2\frac{k_mk_p}{k^2}D_{pn}\right)\tilde{v}_{ln}^{(a)} + E^*N^*k_nD_{nj}\tilde{v}_{lm}^{(a)} + \left(E^*N' + (E^*N^*)' + \frac{1}{2}(E^{*2})'\right)\frac{k_jk_pk_n}{k}D_{pn}\tilde{v}_{lm}^{(a)}$$
$$- E^{*2}k_jD_{mn}\tilde{v}_{ln}^{(a)}\Bigg]d^3k\,d\omega - \frac{1}{2}\epsilon_{ilm}\int\int\Bigg[E^*(N-N^*)\left(D_{ln}-2\frac{k_lk_p}{k^2}D_{pn}\right)\nabla_j\tilde{v}_{mn}^{(s)} + 2E^*(N+N^*)k_j\left(\frac{k_p}{k^2}D_{pn}\nabla_l + \frac{k_l}{k^2}D_{pn}\nabla_p\right.$$
$$\left.-2\frac{k_lk_p}{k^4}D_{pn}(\mathbf{k}\cdot\boldsymbol{\nabla})\right)\tilde{v}_{mn}^{(s)} + [E^{*\prime}N - E^*N' - (E^*N^*)']\frac{k_j}{k}\left(D_{ln}-2\frac{k_lk_p}{k^2}D_{pn}\right)(\mathbf{k}\cdot\boldsymbol{\nabla})\tilde{v}_{mn}^{(s)} - E^{*2}D_{mn}\nabla_j\tilde{v}_{ln}^{(s)}$$
$$-(E^{*2})'\frac{k_j}{k}D_{mn}(\mathbf{k}\cdot\boldsymbol{\nabla})\tilde{v}_{ln}^{(s)}\Bigg]d^3k\,d\omega, \quad \text{(A2)}$$

$$b_{ijk}^{(W)} = \frac{1}{2}\int\int\Bigg\{E^*(N-N^*)(W_i\tilde{v}_{jk}^{(s)} - W_j\tilde{v}_{ik}^{(s)}) - 2E^*N\frac{(\mathbf{k}\cdot\mathbf{W})}{k^2}(k_i\tilde{v}_{jk}^{(s)} - k_j\tilde{v}_{ik}^{(s)}) - E^*N^*\Bigg[\delta_{ik}(W_j\tilde{v}_{ll}^{(s)} - W_l\tilde{v}_{lj}^{(s)}) - 2\left(\frac{k_ik_k}{k^2}(W_j\tilde{v}_{ll}^{(s)} - W_l\tilde{v}_{lj}^{(s)})\right.$$
$$\left.\left. - \frac{k_jk_k}{k^2}(W_i\tilde{v}_{ll}^{(s)} - W_l\tilde{v}_{li}^{(s)})\right)\Bigg] - E^{*\prime}(N-N^*)\frac{k_jk_k}{k}\left(W_i\tilde{v}_{ll}^{(s)} - W_l\tilde{v}_{li}^{(s)} - 2\frac{(\mathbf{k}\cdot\mathbf{W})}{k^2}k_i\tilde{v}_{ll}^{(s)}\right) - E^{*2}(W_i\tilde{v}_{jk}^{(s)} - \delta_{ij}W_l\tilde{v}_{lk}^{(s)})$$
$$+ (E^{*2})'\frac{k_jk_k}{k}(W_i\tilde{v}_{ll}^{(s)} - W_l\tilde{v}_{li}^{(s)})\Bigg\}d^3k\,d\omega, \quad \text{(A3)}$$

$$b_{ijk}^{(D)} = -\int\int\Bigg[E^*N\epsilon_{ijl}\left(D_{lm}-2\frac{k_lk_n}{k^2}D_{nm}\right)\tilde{v}_{mk}^{(s)} + E^*N^*\epsilon_{ijl}\left(D_{km}-2\frac{k_kk_n}{k^2}D_{nm}\right)\tilde{v}_{ml}^{(s)} - E^{*\prime}(N-N^*)\epsilon_{ilm}\frac{k_jk_k}{k}\left(D_{mn}-2\frac{k_mk_p}{k^2}D_{pn}\right)\tilde{v}_{nl}^{(s)}$$
$$+ [E^*N' + (E^*N^*)']\epsilon_{ijl}\frac{k_mk_n}{k}D_{mn}\tilde{v}_{lk}^{(s)} + E^{*2}\epsilon_{ilm}D_{mj}\tilde{v}_{lk}^{(s)} - (E^{*2})'\left(\epsilon_{ilm}\frac{k_jk_k}{k}D_{mn}\tilde{v}_{nl}^{(s)} - \frac{1}{2}\epsilon_{ijl}\frac{k_mk_n}{k}D_{mn}\tilde{v}_{lk}^{(s)}\right)\Bigg]d^3k\,d\omega. \quad \text{(A4)}$$

Again contributions to $b_{ijk}^{(W)}$ and $b_{ijk}^{(D)}$ with δ_{jk} have been dropped.

For the calculation of $a_{ij}^{(W)}$, $a_{ij}^{(D)}$, $b_{ijk}^{(W)}$, and $b_{ijk}^{(D)}$ the gradient tensor $\boldsymbol{\nabla}\bar{\mathbf{U}}$ has been considered as a sum of the two parts expressed by $\mathbf{W}$ and $\mathbf{D}$. Of course, such a calculation can also be carried out without splitting $\boldsymbol{\nabla}\bar{\mathbf{U}}$ in this way. Then a quantity $a_{ij}^{(\nabla U)}$ occurs instead of $a_{ij}^{(W)}+a_{ij}^{(D)}$, and a quantity $b_{ijk}^{(\nabla U)}$ instead of $b_{ijk}^{(W)}+b_{ijb}^{(D)}$. We have written Eqs. (A2) and (A4) such that $a_{ij}^{(D)}$ turns into $a_{ij}^{(\nabla U)}$, and $b_{ijk}^{(D)}$ into $b_{ijk}^{(\nabla U)}$, if on the right-hand sides D_{lm} is replaced by U_{lm}. From these relations for $a_{ij}^{(\nabla U)}$ and $b_{ijk}^{(\nabla U)}$ we can easily derive the relations (A1) and (A2) for $a_{ij}^{(W)}$ and $a_{ij}^{(D)}$ as well as (A3) and (A4) for $b_{ijk}^{(W)}$ and $b_{ijb}^{(D)}$.

APPENDIX B: RELATIONS FOR THE QUANTITIES $\tilde{\alpha}_1^{(W)}, \tilde{\alpha}_1^{(W)}, \ldots, \tilde{\kappa}^{(D)}$

Analogous to the relations (50) we have

$$\tilde{\alpha}_1^{(W)} = (1/120)[\tilde{\eta}^4\tilde{\nu}^3(20\tilde{\eta}-\tilde{\nu}) + 4\tilde{\eta}^2\tilde{\nu}(11\tilde{\eta}^3 + 3\tilde{\eta}^2\tilde{\nu} + 10\tilde{\eta}\tilde{\nu}^2 - 3\tilde{\nu}^3)\omega^2 + (13\tilde{\eta}^4 + 88\tilde{\eta}^3\tilde{\nu} - 20\tilde{\eta}^2\tilde{\nu}^2 + 20\tilde{\eta}\tilde{\nu}^3 + 5\tilde{\nu}^4)\omega^4 - 4\tilde{\eta}(2\tilde{\eta} - 11\tilde{\nu})\omega^6 - 5\omega^8](\tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-2},$$

$$\tilde{\alpha}_2^{(W)} = -(1/240)[\tilde{\eta}^4\tilde{\nu}^3(20\tilde{\eta}-13\tilde{\nu}) + 4\tilde{\eta}^2\tilde{\nu}(3\tilde{\eta}^3 - 11\tilde{\eta}^2\tilde{\nu} + 10\tilde{\eta}\tilde{\nu}^2 + 21\tilde{\nu}^3)\omega^2 - (31\tilde{\eta}^4 - 24\tilde{\eta}^3\tilde{\nu} - 140\tilde{\eta}^2\tilde{\nu}^2 - 20\tilde{\eta}\tilde{\nu}^3 + 15\tilde{\nu}^4)\omega^4 + 4(14\tilde{\eta}^2 + 3\tilde{\eta}\tilde{\nu} - 10\tilde{\nu}^2)\omega^6 - 25\omega^8] \times \tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-2},$$

$$\tilde{\gamma}^{(W)} = -(1/48)[\tilde{\eta}^4\tilde{\nu}^4 + 4\tilde{\eta}^2\tilde{\nu}(2\tilde{\eta}^3 + 2\tilde{\eta}^2\tilde{\nu} + 3\tilde{\nu}^3)\omega^2 + (7\tilde{\eta}^4 + 16\tilde{\eta}^3\tilde{\nu} + 28\tilde{\eta}^2\tilde{\nu}^2 - 5\tilde{\nu}^4)\omega^4 + 4(4\tilde{\eta}^2 + 2\tilde{\eta}\tilde{\nu} - 3\tilde{\nu}^2)\omega^6 - 7\omega^8] \times(\tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-2},$$

$$\tilde{\delta}^{(W)} = (1/12)(\tilde{\eta}^2 - \omega^2)(\tilde{\eta}^2+\omega^2)^{-2},$$

$$\tilde{\kappa}^{(W)} = -(1/30)[\tilde{\eta}^4\tilde{\nu}^2 - \tilde{\eta}^2(23\tilde{\eta}^2 - 12\tilde{\nu}^2)\omega^2 - (12\tilde{\eta}^2 + 5\tilde{\nu}^2)\omega^4 - 5\omega^6](\tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-1},$$

$$\tilde{\alpha}^{(D)} = -(1/120)[3\tilde{\eta}^4\tilde{\nu}^3(4\tilde{\eta} + 3\tilde{\nu}) - 4\tilde{\eta}^2\tilde{\nu}(3\tilde{\eta}^3 - 5\tilde{\eta}^2\tilde{\nu} - 2\tilde{\eta}\tilde{\nu}^2 + 3\tilde{\nu}^3)\omega^2 + (11\tilde{\eta}^4 - 40\tilde{\eta}^3\tilde{\nu} - 12\tilde{\eta}^2\tilde{\nu}^2 - 4\tilde{\eta}\tilde{\nu}^3 - 5\tilde{\nu}^4)\omega^4 - 28\tilde{\eta}\tilde{\nu}\omega^6 + 5\omega^8](\tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-2},$$

$$\tilde{\gamma}^{(D)} = -(1/120)[3\tilde{\eta}^4\tilde{\nu}^3(16\tilde{\eta} - 3\tilde{\nu}) + 4\tilde{\eta}^2\tilde{\nu}(10\tilde{\eta}^3 + 20\tilde{\eta}\tilde{\nu}^2 + 3\tilde{\nu}^3)\omega^2 + (9\tilde{\eta}^4 + 64\tilde{\eta}^3\tilde{\nu} + 52\tilde{\eta}^2\tilde{\nu}^2 + 32\tilde{\eta}\tilde{\nu}^3 + 5\tilde{\nu}^4)\omega^4 + 4(10\tilde{\eta}^2 + 6\tilde{\eta}\tilde{\nu} + 5\tilde{\nu}^2) + 15\omega^8](\tilde{\eta}^2+\omega^2)^{-3}(\tilde{\nu}^2+\omega^2)^{-2},$$

$$\tilde{\beta}^{(D)} = (1/60)[\tilde{\eta}^4\tilde{\nu}^3(10\tilde{\eta} - 3\tilde{\nu}) + 2\tilde{\eta}^2\tilde{\nu}(\tilde{\eta}^3 - 5\tilde{\eta}^2\tilde{\nu} + 8\tilde{\eta}\tilde{\nu}^2 - 3\tilde{\nu}^3)\omega^2 - (7\tilde{\eta}^4 + 16\tilde{\eta}^2\tilde{\nu}^2 - 6\tilde{\eta}\tilde{\nu}^3 - 5\tilde{\nu}^4)\omega^4 - 2(5\tilde{\eta}^2 + \tilde{\eta}\tilde{\nu} - 5\tilde{\nu}^2)\omega^6 + 5\omega^8](\tilde{\eta}^2 + \omega^2)^{-3}(\tilde{\nu}^2 + \omega^2)^{-2},$$

$$\tilde{\kappa}^{(D)} = (1/30)[\tilde{\eta}^4\tilde{\nu}^3(10\tilde{\eta} + 3\tilde{\nu}) + 2\tilde{\eta}^2\tilde{\nu}(\tilde{\eta}^3 + 5\tilde{\eta}^2\tilde{\nu} + 8\tilde{\eta}\tilde{\nu}^2 + 3\tilde{\nu}^3)\omega^2 + (7\tilde{\eta}^4 + 16\tilde{\eta}^2\tilde{\nu}^2 + 6\tilde{\eta}\tilde{\nu}^3 - 5\tilde{\nu}^4)\omega^4 + 2(5\tilde{\eta}^2 - \tilde{\eta}\tilde{\nu} - 5\tilde{\nu}^2)\omega^6 - 5\omega^8](\tilde{\eta}^2 + \omega^2)^{-3}(\tilde{\nu}^2 + \omega^2)^{-2}, \tag{B1}$$

where $\tilde{\eta}$ and $\tilde{\nu}$ stand for ηk^2 and νk^2, respectively.

APPENDIX C: $\Omega\times\mathbf{J}$ EFFECT

For the coefficient $\zeta_1^{(\Omega)}$ defined by (60) we have according to (50),

$$\zeta_1^{(\Omega)} = \frac{64\pi}{15}\int_{k=0}^{\infty}\int_{\omega=-\infty}^{\infty}\frac{(\eta k^2)^2\omega^2}{[(\eta k^2)^2+\omega^2]^2[(\nu k^2)^2+\omega^2]} \times W(k,\omega)k^2 dk d\omega. \tag{C1}$$

Introducing the dimensionless variables $u=(k\lambda_c)^2/q$ and $w=\omega\tau_c$ we find further

$$\zeta_1^{(\Omega)} = \frac{32\pi\tau_c}{15\lambda_c^3}q^{1/2}\int_{u=0}^{\infty}\int_{w=-\infty}^{\infty}\frac{u^{5/2}w^2}{(u^2+\omega^2)^2(P_m^2u^2+w^2)} \times W[(qu)^{1/2}/\lambda_c, w/\tau_c]du dw, \tag{C2}$$

with $P_m=q/p$, and q and p as defined by (53). We may assume that W remains finite everywhere. Clearly $\zeta_1^{(\Omega)}$ always vanishes as $q\to 0$. If p is fixed the same is obvious for $q\to\infty$. With the reasonable assumption that $kW(k,\omega)$ vanishes as $k\to\infty$ we can also in the case of fixed P_m conclude that $\zeta_1^{(\Omega)}$ vanishes as $q\to\infty$.

APPENDIX D: $W\times\mathbf{J}$ DYNAMO

Consider as in RK03 an infinitely extended fluid with a mean shear flow, in a Cartesian coordinate system (x,y,z) given by $\overline{\mathbf{U}}=(0,Sx,0)$ with a constant S, and a superimposed turbulence being homogeneous, isotropic, mirror symmetric, and statistically steady in the limit of vanishing shear. The only nonzero components of $\mathbf{W}$ and $\mathbf{D}$ are then $W_z=S$ and $D_{yx}=D_{yx}=(1/2)S$. Assume further as in RK03 that $\overline{\mathbf{B}}$ does not depend on y. Then the mean-field induction equation (2) together with our results for $\boldsymbol{\mathcal{E}}$ leads to

$$[\partial_t - (\eta+\beta^{(0)})\Delta]\overline{B}_x + \delta S\partial_{zz}^2\overline{B}_y = 0,$$

$$[\partial_t - (\eta+\beta^{(0)})\Delta]\overline{B}_y - S\overline{B}_x - \delta' S\Delta\overline{B}_x = 0,$$

$$\partial_x\overline{B}_x + \partial_z\overline{B}_z = 0 \tag{D1}$$

with

$$\delta = \delta^{(W)} - \tfrac{1}{2}(\kappa^{(W)} - \beta^{(D)} + \kappa^{(D)}),$$

$$\delta' = \delta^{(W)} - \tfrac{1}{2}(\kappa^{(W)} + \beta^{(D)} - \kappa^{(D)}). \tag{D2}$$

The solutions of (6) are

$$\overline{\mathbf{B}} = \hat{\overline{\mathbf{B}}}\exp[\lambda t + i(k_x x + k_z z)] \tag{D3}$$

with some constant vector $\hat{\overline{\mathbf{B}}}$ and

$$\lambda = -(\eta+\beta^{(0)})(k_x^2+k_z^2) \pm |S||k_z|\sqrt{\delta(1-\delta'(k_x^2+k_z^2))}. \tag{D4}$$

We refrain from discussing the case $\delta'(k_x^2+k_z^2)>1$, in which the neglect of higher-order derivatives of $\overline{\mathbf{B}}$ in $\boldsymbol{\mathcal{E}}$ could be questionable. Under this restriction a dynamo can only exist if δ is positive.

According to our results (49) and (B1) for $\delta^{(W)}$, $\kappa^{(W)}$, $\delta^{(D)}$, and $\kappa^{(D)}$ we have

$$\delta = -\frac{\pi}{15}\int_{k=0}^{\infty}\int_{\omega=-\infty}^{\infty}\left(\frac{32(\eta k^2)^2\omega^2}{[(\eta k^2)^2+\omega^2]^2[(\nu k^2)^2+\omega^2]} + \frac{(\eta k^2)^4+12(\eta k^2)^2\omega^2-5\omega^4}{[(\eta k^2)^2+\omega^2]^3}\right)W(k,\omega)k^2 dk d\omega. \tag{D5}$$

Clearly δ grows monotonically with ν. Its maximum, $\delta_{\max}$, is given by

$$\delta_{\max} = -\frac{\pi}{15}\int_{k=0}^{\infty}\int_{\omega=-\infty}^{\infty}\frac{(\eta k^2)^4+12(\eta k^2)^2\omega^2-5\omega^4}{[(\eta k^2)^2+\omega^2]^3} \times W(k,\omega)k^2 dk d\omega. \tag{D6}$$

With an integration by parts with respect to ω this turns into

$$\delta_{\max} = \frac{\pi}{15}\int_{k=0}^{\infty}\int_{\omega=-\infty}^{\infty}\frac{(\eta k^2)^2+5\omega^2}{[(\eta k^2)^2+\omega^2]^2}\frac{\partial W(k,\omega)}{\partial\omega}k^2\omega dk d\omega. \tag{D7}$$

It seems reasonable to assume that $\omega\partial W/\partial\omega \leq 0$. Then $\delta_{\max}$ can never be positive. Consequently δ is never positive, and a $\mathbf{W}\times\mathbf{J}$ dynamo as considered above cannot work. This conclusion applies independent of specific ansatzes like (51) or (52).

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